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# Triggering Long-Short Trades on Indexes

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***Abstract*— This paper analyzes the predictivity and return performance of the Barmish-Iwarere feedback trading algorithm described in [1]. In the first part of the paper, we study the trade triggering algorithm using either an Ito process model, or real data from indexes and ETFs. It is shown through hypothesis testing that the trigger provides mixed results in predicting the sign of the single trade, for both the Ito process and real indexes. However, we show empirically that the trigger is sufficiently good in identifying a trend, while it fails in detecting side movements. In the second part of the paper, we analyze the effect of controller parameters under various market circumstances. The efficiency of a pre-optimization on historical data appears controversial. Some modifications are experimented, with the objective of improving the returns. In particular, the trigger is modified to detect anomalous falls during a rising trend using the estimated volatility. The resulting system is then tested with other indexes, commodities and interest rates.**

***Index Terms*— Trading system; trigger; feedback controller; long-short trades.**

## I. INTRODUCTION

A mathematical model which is frequently used to approximate the behavior of real markets is the Ito process [2], that is a Brownian motion with drift. A Brownian motion (also known as Wiener process) has three properties:
- is a Markov process: the probability distribution for all future values of the process depends only on its current value;
- has independent increments: the probability distribution for the change in the process over any time interval is independent of any other (non overlapping) time interval;
- changes over any finite interval of time are normally distributed.

An Ito process is described by the equation:

$$dS = a(S,t)dt + b(S,t)dz \tag{1}$$

where, $dz$ is the increment of a Wiener process, $a(S,t)$ is the drift parameter, $dz$ can be represented as $dz = \epsilon_t\sqrt{dt}$, where $\epsilon_t$ is a normal random variable with zero mean and unit standard deviation. A special case of (1) is the geometric Brownian motion with drift, here $a(S,t) = \mu S$, $b(S,t) = \sigma S$, where $\mu$ and $\sigma$ are constant, and the equation becomes:

$$dS/S = \mu dt + \sigma\epsilon_t\sqrt{dt} \tag{2}$$

so that the instantaneous rates of return $dS/S$ are normally distributed, as confirmed roughly by data analysis for real returns of stocks, and, for the Ito's lemma, the increment $d(lnS)$ will be [2]: $d(lnS) = (\mu - \frac{1}{2}\sigma^2)dt + \sigma\epsilon_t\sqrt{dt}$.

Equation (2) is an approximated model which is often used to simulate stock prices, being the differences with real data usually negligible for high frequency data. Mean reversion is one of the observed deviation from the model [3]: it is the tendency of stock prices to be attracted towards their long term mean; interest rates and raw commodities exhibit a mean reverting behavior. The simplest model for a mean reverting process is:

$$dS = \eta(\overline{S} - S)dt + \sigma\epsilon_t\sqrt{dt} \tag{3}$$

where $\eta$ is the rate of reversion, and $\overline{S}$ is the level to which $S$ tends to revert. It should be noticed that this process satisfies the Markov property, but does not have independent increments.

Some studies have shown the tendency of stock prices to overreact [3], investors are subject to waves of optimism and pessimism that cause prices to deviate systematically from their fundamental values and later to exhibit mean reversion, in such cases the "contrarian" strategy could be profitable, this strategy has been used in the experiments.

## II. THE TRADING SYSTEM

The trading system we analyze is composed by a trigger and a controller [1], the trigger gives the signal for entering or exiting a trade, the controller modulates the amount invested with the objective of improving the return. The system was tested at first with simulations of an Ito process. The model used for generating the sequences is equation (2), hence the price $S(k+1)$ is given by: $S(k+1) = (1 + \mu\Delta t + \sigma\epsilon(k)\sqrt{\Delta t})S(k)$, where $\Delta t$ is the time interval between potential trades measured in years, which is set to a one day interval $\Delta t = 1/252$, being a trading year composed by around $252$ days. $\mu$ is the annualized drift of the stock, $\sigma$ the annualized volatility of the stock, and $\epsilon(k)$ is a normal random variable with zero mean and unit standard deviation. An estimation $\hat{\mu}$ of $\mu$ and $\hat{\sigma}$ of $\sigma$ is computed from $n$ simulated or real market data, then $\hat{\sigma}$ is used to build a confidence interval for $\hat{\mu}$.

The one period return used to obtain the estimation is: $\rho(k) = S(k+1)/S(k) - 1$. For real market data $S(k)$ is the closure price of the stock. Finally the estimates are:

$$\hat{\mu}(k) = \frac{1}{n\Delta t}\sum_{i=1}^{n}\rho(k-i)$$

$$\hat{\sigma^2}(k) = \frac{1}{n-1}\sum_{i=1}^{n}(\rho(k-i)/\Delta t - \hat{\mu})^2,$$

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and a confidence interval on $\hat{\mu}$ is given by

$$[L(k), U(k)] = [\hat{\mu}(k) - \frac{t_{\frac{\alpha}{2},n-1}\hat{\sigma}(k)}{\sqrt(n)}, \hat{\mu}(k) + \frac{t_{\frac{\alpha}{2},n-1}\hat{\sigma}(k)}{\sqrt(n)}],$$

where $1-\alpha$ is the chosen confidence level, $t_{\alpha/2,n-1}$, the critical value from the Student $t$-distribution with $n-1$ degrees of freedom. With daily data there is no need to use the exact formulas from the Black-Scholes model to compute $\hat{\mu}$ and $\hat{\sigma}$ [4]. Once the interval $[L, U]$ is computed, the rule to trigger a trade is:

- if the lower extreme of the confidence interval $L$ satisfies $L \geq 0$, a long trade is triggered;
- if the upper upper extreme $U$ satisfies $U \leq 0$, a short trade is triggered;
- for the case when $L < 0 < U$, no trade is triggered.

If the trigger returns a signal of trade, the controller determines the amount to invest. For instance, if at time $k^*$ after a period of no trade there is a long signal, then a long trade begins. Assuming no commissions and an account value $V(k^*)$, the initial investment is: $I(k^*) = \gamma_0 V(k^*)$, $0 < \gamma_0 \leq 1$. How to choose a suitable $\gamma_0$ is currently an open problem [1]; in this paper, we have used a training sequence of previous data and the Kelly criterion, as exposed in the following sections.

Once a long trade is entered, as the stock price goes from $S(k^*)$ to $S(k^*+1)$ with return $\rho(k^*) = S(k^*+1)/S(k^*)-1$, the account value becomes:

$$V(k^*+1) = V(k^*) + \rho(k^*)I(k^*)$$

and the amount invested is tuned with the rule:

$$I(k^*+1) = [1 + K\rho(k^*)]I(k^*), \quad (4)$$

where $K$ is the feedback gain: $K = 1$ is a "buy and hold" strategy until the position is open, that is the trade is started with the initial amount and no action is performed, the amount invested varies accordingly to the return of the stock. Otherwise if $K \neq 1$, for example $K > 1$, supplemental money is invested in case of profit, or disinvested in case of loss. As the trade evolves the amount invested is updated according to (4). However a limit in the exposure is set via a saturation condition:

$$I(k^*+j) = \gamma_{max} V(k^*+j) = I_{max}; \quad j > 0.$$

Therefore, the actual investment evolves according to:

$$I(k^*+j+1) = \min\{[1 + K\rho(k^*+j)]I(k^*+j), I_{max}\}.$$

For short trading the same formulas hold, but $-1 \leq \gamma_0 < 0$, so $I(k^*) < 0$, and $K < 0$, therefore in this work short trading means to buy an inverted stock. Since we use ETFs in our simulations, inverted ETFs (or ETF short) will be the stocks for going short, for example in trading the MSCI Emerging Markets Index, EEM will be used for a long position, and EUM for a short one; unfortunately it does not exist an inverted ETF for every index used in this work.

### A. The Kelly Criterion

As seen in the previous section, one of the open problem of the trading system is the value to assign to $\gamma_0$, that is what fraction of capital to initially allocate to the risky investment and how much to keep in cash. A possible choice can be the *optimal Kelly fraction* [5] (also known as Latané strategy [6]). The goal of this strategy is to maximize the growth of the capital over the long term. Supposing to invest in $m$ trials and that the amount invested is $I(k) = \gamma_0 V(k)$, then the capital after $m$ trials is:

$$V_m = V_0(1 + \gamma_0 g)^S (1 - \gamma_0 l)^F$$

where $S$ and $F$ are the number of successes and failures, $S + F = m$, $g$ is the gain and $l$ is the loss during a single trial. If $0 < l < 1$, it is not possible to lose more than the amount invested, and if $0 < \gamma_0 < 1$, $Pr(V_m = 0) = 0$ also if $l = 1$.
Since

$$e^{m\ln(\frac{V_m}{V_0})^{1/m}} = \frac{V_m}{V_0}$$

$$\ln(\frac{V_m}{V_0})^{1/m} = \frac{S}{m}\ln(1 + \gamma_0 g) + \frac{F}{m}\ln(1 - \gamma_0 l)$$

the last quantity measures the exponential rate of increase per trial: for having growth this has to be greater than zero. The Kelly criterion maximizes the expected value:

$$E\{\ln(\frac{V_m}{V_0})^{1/m}\} = E\{\frac{S}{m}\ln(1 + \gamma_0 g) + \frac{F}{m}\ln(1 - \gamma_0 l)\}$$

$$p\ln(1 + \gamma_0 g) + q\ln(1 - \gamma_0 l),$$

where $p$ is the probability of gain, $q = 1-p$ is the probability of loss. The unique optimal fraction is hence:

$$\gamma_0 = \frac{pg - ql}{gl} = \frac{p(l+g) - l}{gl}; \quad pg - ql > 0. \quad (5)$$

For the optimal value, the expected growth factor per trial is:

$$p\ln p + q\ln q + p\ln(1 + g/l) + q\ln(1 + l/g).$$

It can be shown that the mean first passage time to arbitrary large wealth targets is minimized, however the strategy is very aggressive due to the volatility of the total return. Even if all the parameters are exact, that is in absence of estimation and chance errors, there is a very high volatility of wealth levels, in fact the expected growth factor times $m$, gives the natural logarithm of the median wealth, but the distribution is dispersed [6].
In presence of estimation errors, owing to a high sensitivity to the parameter values, either in the fraction, or in the return, wrong estimates impact heavily on the actual return. Investing a fraction greater than the optimal one may bring to bankruptcy, so it is common to use a lower fraction, typically a half, but the risk may be high anyway. Table I shows the optimal Kelly fraction for values of gains and losses typical of a betting system ($l = 100\%$). The bettor knows with certainty $l$ and $g$, he has only to estimate his probability of success $p$, hence he chooses the fraction to invest within

TABLE I
OPTIMAL KELLY FRACTION (L=100%)

| p of win\Gain | 100% | 200% | 300% | 400% | 500% |
|---|---|---|---|---|---|
| 20% | | | | | 4% |
| 30% | | | 7% | 13% | 16% |
| 40% | | 10% | 20% | 25% | 28% |
| 50% | | 25% | 33% | 38% | 40% |
| 60% | 20% | 40% | 47% | 50% | 52% |

one column of the table, where the variation of the values is important but not dramatic.

For the stock investor all the parameters $p$, $g$ and $l$ are unknown and he must estimate them. In table II the optimal fraction is tabulated versus the gain and loss typical of a single day trading system, assuming an optimistic probability of gain $p = 60\%$. The dispersion is huge (values greater than one imply using leverage) passing from "no trade" to a high leveraged trade. Only for arbitrages the value of the gain and the loss is almost certain.
For stock trading, where there is a continuum of outcomes, it can be found that the optimal Kelly fraction to be used is: $\gamma_0 = \frac{\hat{\mu}-r}{\hat{\sigma}^2}$ , where $r$ is the risk-free interest rate, however this formula depends on the square of the estimated volatility, and so the fraction undergoes high variations. The investor should dynamically reallocate his resources as $\gamma_0$ changes over time because of fluctuations in the forecasts, but it could be a very risky game.

TABLE II
OPTIMAL KELLY FRACTION (P=0.6)

| Gain\Loss | 0.50% | 1.00% | 1.50% | 2.00% | 2.50% |
|---|---|---|---|---|---|
| 0.5% | 4000% | | | | |
| 1.0% | 8000% | 2000% | 0% | | |
| 1.5% | 9333% | 3333% | 1333% | 333% | |
| 2.0% | 10000% | 4000% | 2000% | 1000% | 400% |
| 2.5% | 10400% | 4400% | 2400% | 1400% | 800% |

The high yield bonds investor is in an intermediate situation, the optimal Kelly fraction for a 5 year investment is shown in table III, $1-l$ is the recovery percentage, $1+g$ the total amount in excess of a risk-free bond of similar maturity, $q$ the probability of default in the period. In November 2010 the ML Euro High Yield spread was about 500 basis point, so the gain is roughly known, being unknown only the reinvestment rate of the proceeds, here assumed to be zero.

TABLE III
OPTIMAL KELLY FRACTION (G=25%)

| Default prob.\Recovery perc. | 50% | 40% | 30% | 20% | 10% |
|---|---|---|---|---|---|
| 25% | 50% | 25% | 7% | | |
| 20% | 80% | 53% | 34% | 20% | 9% |
| 15% | 110% | 82% | 61% | 46% | 34% |
| 10% | 140% | 110% | 89% | 73% | 60% |
| 5% | 170% | 138% | 116% | 99% | 86% |

## III. THE PREDICTIVITY OF THE TRIGGER

In order to apply the Kelly criterion to the parameter $\gamma_0$, we investigated at first if the estimated return $\hat{\mu}$ is "near" the real return $\rho$. Unfortunately either for simulated or real stock prices, the correlation coefficient between $\hat{\mu}$ and $\rho$ is low, from about 7% to 13%.

We next studied if the sign of the predicted return is meaningfully predicted by the trigger. The following random variable is defined: $X = \text{Sign}(\rho)f$, where $f$ is the flag of trading assuming value 1 for a long trade, -1 for a short trade, and 0 for no trade. We assume as null hypothesis that the trigger is unable to predict the sign of the return, and it behaves like a source emitting symbols $\{-1, 0, 1\}$ with probabilities $\{q, r, p\}$. Given $N_+$ the number of positive or zero returns, $N_-$ the number of negative returns, $N = N_+ + N_-$, if the output of the trigger is independent from the sign of the return, then:

$$E\{X\}_N = E\{\frac{p(N_+(1) + N_-(-1)) + r(N_+ + N_-)(0)+}{N} \frac{+q(N_+(-1) + N_-(1))}{N}\}$$

$$E\{X\}_N = pp' - qp' - pq' + qq' = (p-q)(p'-q')$$

where $E\{N_+/N\} = p'$, and $E\{N_-/N\} = q' = 1 - p'$.
In the same manner for $E\{X^2\}$:

$$E\{X^2\}_N = pp' + qp' + pq' + qq' = (p+q)(p'+q') = p+q$$

finally an upper bound for the variance is:

$$\sigma_N^2 = E\{X^2\}_N - E^2\{X\}_N = p+q-(p-q)^2(p'-q')^2 \leq 1$$

because $p+q \leq p+q+r = 1$ and $(p-q)^2(p'-q')^2 = 0$ if $p = q$ or $p' = q'$. Fixing the significance level to 99%, $m^*$, the margin of error to 1%, then $N$, the number of simulated trades, has to be at least 66.349: $N = \lceil(\sigma z/m^*)^2\rceil$. For every set of parameters 5 simulations of different seed were run involving 67.000 trials, the high number of trials justifies the assumption of normality for $E\{X\}$ [7]. The results show a direct dependence on the drift to volatility ratio, and inverse dependence on $n$. The null hypothesis is falsified, although the predictivity is low. In table IV are reported the number of predicted signs minus the unpredicted ones for some values of the parameters.

Observing all the data available it can be seen that the confidence level $1-\alpha$ is not an important factor, the number of positive trade varies slowly with it, at least in the range $[0.80, 0.99]$. Moreover, the dependence appears a bit erratic. However a better choice seems to be from 0.8 to 0.9. Nevertheless, as shown in the following paragraph, for volatile equities also a little number of good trades may lead to important differences. It should be remarked that the values of the window width $n$ and of the confidence level are kept fixed over all the trading time, without adapting them to the particular moment.

Using real indexes predictivity seems to improve, as it can be seen from table V for the S&P500 Index from 1950

TABLE IV

NUMBER OF PREDICTED MINUS UNPREDICTED RETURN SIGNS

| n | Pred-Unpr | % | $\hat{E}\{X\}_N$ | $\mu/\sigma$ y. |
|---|---|---|---|---|
| 60 | 5,585 | 9.1% | 0.6% | 0.5 |
| 110 | 4,084 | 6.6% | 0.7% | 0.5 |
| 160 | 3,415 | 5.5% | 0.9% | 0.5 |
| 210 | 3,341 | 5.4% | 1.0% | 0.5 |
| 60 | 6,399 | 10.3% | 2.0% | 1.0 |
| 110 | 4,782 | 7.7% | 2.6% | 1.0 |
| 160 | 4,267 | 6.7% | 2.9% | 1.0 |
| 210 | 3,822 | 6.0% | 3.3% | 1.0 |
| 60 | 7,918 | 12.4% | 6.8% | 2.0 |
| 110 | 7,272 | 11.2% | 8.3% | 2.0 |
| 160 | 7,467 | 11.4% | 9.0% | 2.0 |
| 210 | 6,889 | 10.4% | 9.5% | 2.0 |

to February 2010 and the EEM ETF replicating the MSCI Emerging Markets index, from April 2003 to February 2010.

However, for the choice of $\gamma_0$, the predictive power of the trigger does not appear sufficiently strong to use the Kelly criterion, which is so sensitive to the parameters estimates.

TABLE V

PERCENTAGE OF PREDICTED MINUS UNPREDICTED RETURN SIGNS

| Index | 60 | 110 | 160 | 210 | $\mu/\sigma$ y. |
|---|---|---|---|---|---|
| S&P 500 1950-2010 | 10.9% | 9.8% | 8.4% | 7.8% | 0.53 |
| S&P 500 2003-2010 | 10.3% | 11.3% | 8.9% | 7.7% | 0.25 |
| EEM 2003-2010 | 13.4% | 10.1% | 9.1% | 8.7% | 0.70 |

After investigating the predictive capability of the trigger for a single day of trading, we studied its capacity to identify a trend through an empirical study. Figure 1 shows the behavior of the trigger in detecting the trends for the EEM ETF. With $n = 60$, the trigger identifies reasonably well the trends, however it lacks precision in marking periods of side movements, where a signal of "no trade" would be desirable.

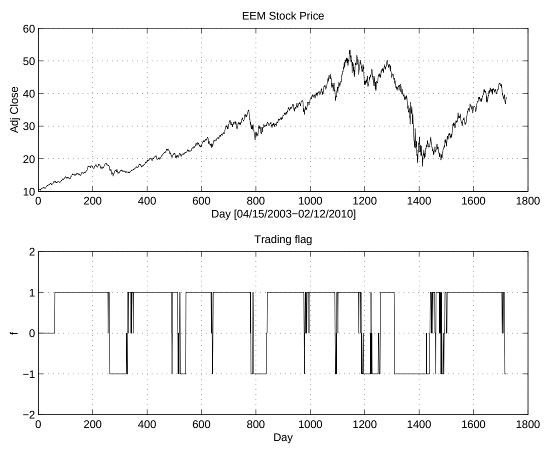


Fig. 1. EEM and Trading flag, f=1 long, f=-1 short, f=0 no trade; n=60

Finally, a return comparison using markets data was conducted between the Barmish system with gain $K = 1$, $\gamma_0 = 1$ and $\gamma_{max} = 1$, and a system based on a classical technical analysis indicator [8], a moving average of equal width. The first one outperforms the moving average strategy almost always, the results for EEM are in table VI.

TABLE VI

BARMISH SYSTEM VS MOVING AVERAGE, FINAL TRADED VALUES

| n | Barmish | Mov. Avg. |
|---|---|---|
| 60 | 23.3 | 17.6 |
| 90 | 31.8 | 22.6 |
| 120 | 18.6 | 27.9 |
| 150 | 38.1 | 21.7 |
| 180 | 44.8 | 22.8 |

## IV. OPTIMIZING THE PARAMETERS AND THE TRIGGER

Simulations on the Ito process and real markets data were run at first to study the sensitivity of the final traded value to the parameters, and then to try an optimization of them, moreover some modifications were introduced in order to take into account the results about the trigger described in the preceding paragraph. Transaction costs and bid-ask spreads were not considered.

### A. *The window width $n$ and the confidence level*

As previously seen the window width is very important for the process of triggering. $n$ in the range of 50 - 80 captures well the trends for volatile equities, like a single stock or an exotic index, but side movements usually are not detected, so the system may incur in important losses, particularly if the controller gain $K$ is greater than one. Increasing the window width $n$ reduces the jitter and enhances the results, even if sometimes the results are inferior to a "buy and hold" strategy.

TABLE VII

EEM FINAL TRADED VALUE FOR CONFIDENCE LEVEL AND WINDOW WIDTH (EEM FINAL VALUE 38.4; K=0.2)

| n | 80% | 90% | 95% | 99% |
|---|---|---|---|---|
| 60 | 28.0 | 26.4 | 26.7 | 26.6 |
| 70 | 32.9 | 36.4 | 40.0 | 40.4 |
| 80 | 47.4 | 48.9 | 50.4 | 44.0 |
| 90 | 35.2 | 37.3 | 43.2 | 40.8 |
| 100 | 32.4 | 36.8 | 37.5 | 38.8 |
| 110 | 33.2 | 34.8 | 34.7 | 38.4 |
| 120 | 20.7 | 21.9 | 26.1 | 33.2 |
| 130 | 25.8 | 25.9 | 29.4 | 32.7 |
| 140 | 56.3 | 51.2 | 37.5 | 35.8 |
| 150 | 46.4 | 45.1 | 45.6 | 45.5 |
| 160 | 47.3 | 45.1 | 40.8 | 47.5 |
| 170 | 43.5 | 45.9 | 48.1 | 42.4 |
| 180 | 49.7 | 49.3 | 47.8 | 44.9 |

In pictures 2 - 5 the first subplot shows the value of S&P500 and EEM from April 2003 to February 2010, and the subplots below the values $V(k)$ of the trading system for different values of $n$, and for $K = 0.2$, $\alpha = 0.1$. In order to compare the series it is supposed that the entry value $V(0)$ is equal to the value of the index in the first day of trading.

It can be seen very well that 180 days is a good choice for $n$, but not the best one, which is 140, as shown in table VII, however the final traded value varies abruptly with $n$ (e.g. $n$ from 120 to 140).
Moreover even if $1 - \alpha$ rarely is an important factor, i.e. the number of triggered trades is almost insensitive to it, the outcomes during high volatility periods dramatically change. Finally another fact emerges from the observation of the subplots, the optimal $n$ changes during the time of trading, it is as if the financial signal varies his time constant: for EEM the last rising trend is well caught by the 90 days system, while the collapse of fall 2008 is matched for $n = 180$ and almost ignored for $n = 120$, while for S&P500 the 90 days system is unable to exploit both the trends, so the usefulness of a pre-optimization on the last $n$ data [1] seems doubtful.

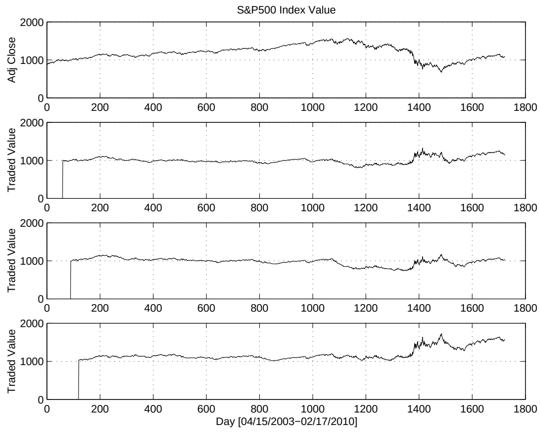


Fig. 2. S&P500 - Index value and traded values, n=60,90,120

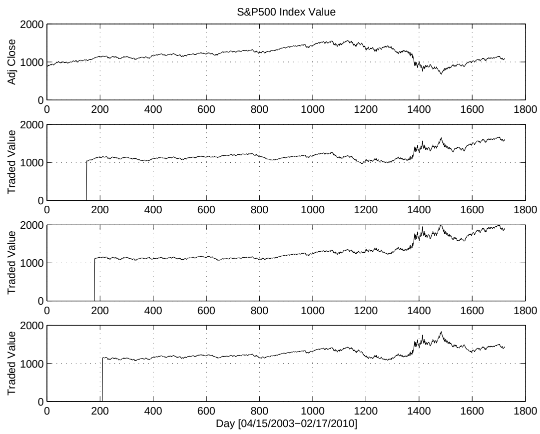


Fig. 3. S&P500 - Index value and traded values, n=150,180,210

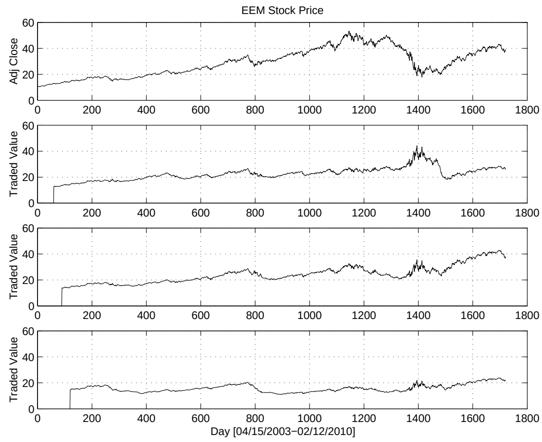


Fig. 4. EEM - ETF quote and traded values, n=60,90,120

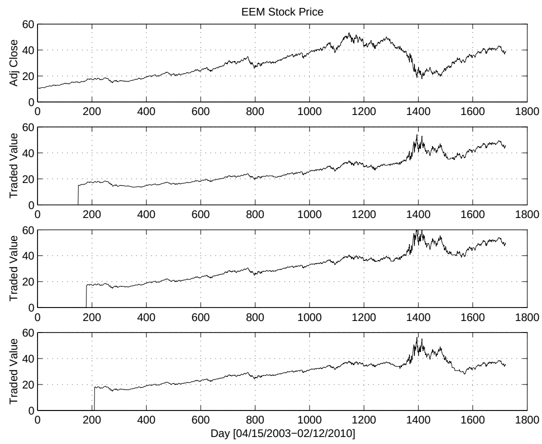


Fig. 5. EEM - ETF quote and traded values, n=150,180,210

### B. The gain K and $\gamma_0$

The preceding results were found with $K = 0.2$, greater values of $K$ often decreases the return, but this is true only for the last years. For S&P500 a $K$ near to one was found optimal in previous times. A $K$ greater than one can cause huge losses during financial turbulences, for firm shares like investment banks during the 2008 crisis. Owing to the low predictivity of the trigger over a single day it is very likely to encounter a sequence where a high portion of the capital is invested in "bad days" and almost nothing in "good days" leading to early bankruptcy. It was investigated also if there is a convenience to use two gains, $K_L$ for long positions, $K_S$ for short positions, the motivation liying in the different behavior of financial markets during rising and falling cycles summarized by the expressions *"Up a staircase, down an elevator"* and *"The bull walks up the stairs and the bear jumps out the window."* However, no evidence was found to introduce two gains, but improvements were obtained modifying the trigger only with falling markets. Finally, the best results were obtained with a value of $\gamma_0 = \pm 1$, in index trading it seems the optimal choice, being relatively small the risk of a huge loss, in practice the purchase of options out of the money will hedge the position partially.

### C. The integrative controller and the reverting to the mean process

The trigger is partially able to predict a single positive trade, but detects pretty good the trends, so it was introduced an integrative part in the controller. The amount invested for a trade beginning at time $k^*$ is modulated by the algebraic sum of the preceding $l$ gains. However, there is no advantage to use this technique. After this simulation, we tested taking $K > 0$ during a short trade, i.e. a rising amount was invested proportionally to losses. Surprisingly, the final traded value increased for some stocks, most of the positive trades happened near the minimum following the 2008 crisis confirming the mean reversion of stock prices [3] and the profitability of the ”contrarian” strategy in some occasions.

### D. Improving the trigger

We have attempted to use the estimated volatility to detect anomalous changes in the quotations. Within a frame of long trading, the relative deviation of the current price versus the maximum in the frame is compared with a multiple of the volatility. If the deviation exceeds the bound then a short trade is triggered:

- if during a long trade beginning at $k = k^*$

$$\frac{S(k)}{S_{max}} - 1 < -z_L \hat{\sigma}\sqrt{\Delta t}; \quad S_{max} = \max_{k \geq k^*}[S(k)]$$

  then a short trade is triggered.

As usual $z_L$ is set out in order to maximize the final value. We have tried also to detect anomalous deviations within a short trading, but the rule is not effective.

TABLE VIII
FINAL TRADED VALUES (FINAL VALUES: EEM 38.4, S&P500 1,100)

| | EEM | | S&P500 | |
|---|---|---|---|---|
| n | Barm. | Mod. Barm. | Barm. | Mod. Barm. |
| 60 | 23.3 | 23.2 | 1,074 | 1,135 |
| 70 | 32.5 | 32.7 | 1,019 | 1,121 |
| 80 | 43.0 | 47.6 | 1,037 | 1,138 |
| 90 | 31.8 | 39.5 | 997 | 1,025 |
| 100 | 32.2 | 37.1 | 1,163 | 1,147 |
| 110 | 31.2 | 36.5 | 1,472 | 1,511 |
| 120 | 18.6 | 22.0 | 1,505 | 1,523 |
| 130 | 21.2 | 27.0 | 1,337 | 1,337 |
| 140 | 41.8 | 51.5 | 1,284 | 1,299 |
| 150 | 38.1 | 37.1 | 1,514 | 1,582 |
| 160 | 38.0 | 32.0 | 1,543 | 1,551 |
| 170 | 37.1 | 31.8 | 1,492 | 1,485 |
| 180 | 44.8 | 45.3 | 1,783 | 1,710 |
| 190 | 41.5 | 44.1 | 1,632 | 1,648 |
| 200 | 34.8 | 37.3 | 1,372 | 1,333 |
| 210 | 34.6 | 37.1 | 1,318 | 1,275 |

Table VIII shows the final traded values for EEM and S&P500 with the Barmish trading system and with the modified trigger. In the simulation $K = 1$ (the shares are bought and sold only at triggering times, reducing dramatically the penalty for transactions costs and bid-ask spreads), $\alpha = 0.1$, $z_L = 4.5$. In most cases results improve, especially for the volatile EEM. The period of trading is April 2003 - February 2010.

## V. TESTING WITH OTHER STOCKS

After the optimization, a series of tests were performed with other indexes and also with stocks for which the system was not designed: commodities and interest rates, which are better described by a mean reverting process like (3). At first were tested the analogues for Europe of the S&P500 and of the MSCI Emerging Markets index: the S&P350 Europe and S&P Emerging Europe index, replicated by the ETFs IEV and GUR. It can be objected that EEM and GUR holds some equities in common, Gazprom for example, it is true only in little part but it is not essential: many stock indexes are strongly correlated, however trading systems, like the one proposed, perform differently with them. The period under study is from the end of March 2007 to the first half of November 2010, so partially overlaps the one in the previous simulation. The values of the parameters are the same: $\alpha = 0.1$, $K = 1$, $z_L = 4.5$. Comparing table IX with table VIII it can be seen that the results found before are substantially confirmed:
- the modified trigger performs better for GUR, more volatile than IEV;
- the return depends strongly on $n$ and it shows roughly the same dependence with $n$; this matter needs further investigation;
- the traded value is almost always better than the ETF value, so the systems works better for GUR and IEV than for EEM and S&P500.

TABLE IX
FINAL TRADED VALUES (FINAL VALUES: GUR 46.7, IEV 38.4)

| | GUR | | IEV | |
|---|---|---|---|---|
| n | Barm. | Mod. Barm. | Barm. | Mod. Barm. |
| 60 | 35.6 | 46.1 | 50.6 | 56.2 |
| 70 | 74.4 | 81.3 | 66.9 | 75.8 |
| 80 | 96.1 | 92.0 | 52.3 | 56.4 |
| 90 | 54.0 | 57.2 | 44.0 | 44.7 |
| 100 | 66.0 | 78.9 | 55.8 | 54.3 |
| 110 | 64.5 | 67.1 | 64.7 | 65.6 |
| 120 | 70.3 | 71.0 | 67.1 | 67.6 |
| 130 | 52.7 | 55.0 | 62.3 | 61.7 |
| 140 | 101.8 | 102.7 | 73.7 | 69.4 |
| 150 | 63.5 | 73.7 | 89.7 | 84.5 |
| 160 | 47.2 | 57.7 | 61.6 | 57.7 |
| 170 | 52.3 | 63.0 | 55.2 | 50.8 |
| 180 | 63.5 | 78.2 | 49.8 | 45.9 |
| 190 | 55.0 | 65.6 | 40.9 | 36.9 |
| 200 | 54.9 | 68.2 | 30.0 | 29.4 |
| 210 | 73.0 | 81.5 | 31.0 | 30.8 |

As before, the lack of performance occurs sometimes for nearby values of $n$: picture 6 shows the GUR stock price and the traded values for $n = 130$ and 140. In the first case the two periods of side movements before the great fall are misinterpreted by the trigger. This fact is highlighted also in picture 1. The better short trades for $n = 140$ are not determinant, they are counterbalanced by the undetected bounce back. Almost the same happens for IEV, the two periods of size movements are undetected, and there is a delay in recognizing the decreasing trend.

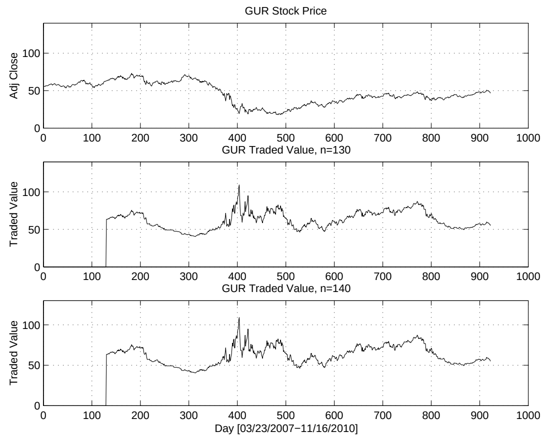


Fig. 6. GUR - ETF quote and traded value, n=130,140, modified trigger

The similar returns dependence on $n$ between GUR and EEM, IEV and S&P500 is probably due to the partial overlapping of the trading periods, in fact different results come out considering different periods: for IEV in the period from August 2000 to March 2004 the best values for $n$ are 120 and 130, instead of 140 and 150; for S&P500 from March 1960 to December 1966 the best values are 90 and 100, instead of 180 and 190. The same thing has been found also for other indexes.
So, as stressed in the preceding section, the optimal $n$ changes relatively fast, however usually there is a range of suboptimal values, this range changes too, but more slowly.

The system was then tested with commodities and interest rates, the mathematical models are different for them, however as the trading system has shown a different behavior with stock indexes, it has believed appropriate to try. The commodities considered were: gold, natural gas, S&P GSCI Commodities Index, for interest rates was chosen the Barclays Capital U.S. 20+ Year Treasury Bond index, among the ETFs replicating them there are: GLD, UNG, GSP and TLT.

In picture 7 it can be seen how UNG, GSP and TLT show the typical empirical behavior described in literature [9]: the tendency to move between levels. Such patterns should be easily detectable by the system. However, occasionally there are disturbing movements for the trigger which are significantly larger and seems to be jumps as for TLT.
Table X shows the results of the simulations. "Extra return" is defined as the relative deviation between the final traded value and the final stock value, $V(k_{fin})/S(k_{fin}) - 1$. Extra returns are very good for natural gas and the commodities index, for natural gas they are extreme for all the values of $n$, the reason is that the quote has collapsed relative to the maximum, and the decreasing trend is correctly detected by the modified trigger. However, before the beginning of the fall the traded values are near the ETF price for almost all the values of $n$. Instead jumps, volatility and similar levels are responsible of the loss for TLT.

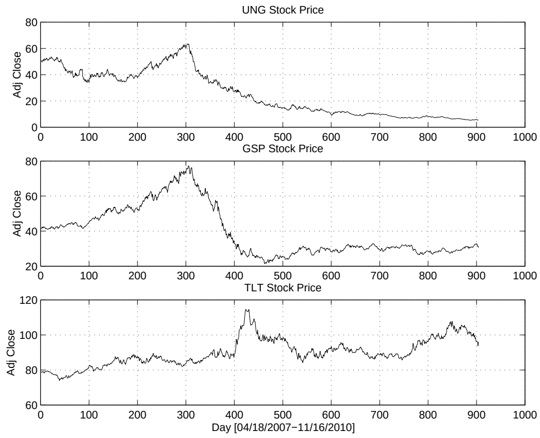


Fig. 7. UNG, GSP and TLT - ETF quote

TABLE X
TRADING SYSTEM RESULTS

| Ticker | System type | n | Extra return | Period |
|---|---|---|---|---|
| UNG | Mod Barm | 110 | 30.97 | 04/18/07 - 11/16/10 |
| GSP | Mod Barm | 60 | 2.81 | 04/18/07 - 11/16/10 |
| TLT | Barm | 90 | 0.04 | 04/18/07 - 11/16/10 |
| GLD | Barm | 150 | -0.25 | 11/18/03 - 11/16/10 |

Unfortunately GLD is one of worst stock, in picture 8 is plotted the trading flag and the results of the trading, the loss is caused by the wrong short trades at the onset of the volatile rising trend around day 1000, moreover after that there is a period of uncertain trades.

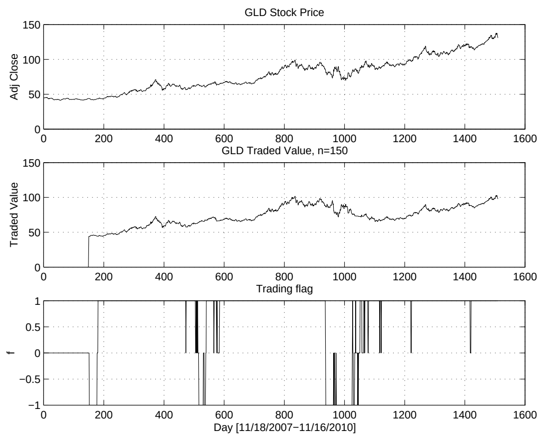


Fig. 8. GLD - ETF quote, traded value and trading flag

## VI. CONCLUSIONS

### A. Conclusions

This paper has analyzed several variations around the Barmish and Iwarere trading system. The system is composed by a trigger and a controller: it has been found that the trigger shows some predictivity and the outcomes of the whole system are usually good with moderate volatility indexes, like S&P500, and less good with more volatile indexes, especially with firm shares.

Many simulations were run to optimize parameters. The window width $n$ is the most important. However, an optimization with a short learning sequence is not opportune, in fact the optimal $n$ changes and it cannot be estimated preemptively, so it is better to use a suboptimal value but such to guarantee robustness.

The confidence level $1-\alpha$ is not determinant for non volatile stocks and it is difficult to optimize. Moreover the controller gain $K$ has to be kept low or moderate, $K \leq 1$.

Eventually, we introduced a change in the trigger in order to detect the inversion of a rising trend. The attempt was successful, especially for volatile indexes. The final versions were tested also with commodities and interest rates indexes.

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