## Auction Pricing Mechanism of Construction Land Quota Based on Preference

#### Yang Deng

Abstract—In order to study the impact of the location preference of construction land on the auction pricing strategy of construction land quota, this study constructs a first-price auction and two-price auction model for bidders participating in the construction land quota and construction land auction based on the sequential auction theory. Therefore, the role of preference is discussed, and finally the conclusion is verified by an example. The results show that under the two auction modes, preference has a positive impact on the bidding strategy of construction land quota and farmers' expected income. Compared with sequential first-price auction, the bidding strategy of sequential second-price auction is more competitive, and the quota equilibrium bidding price and farmers' income obtained are higher.

*Index Terms*—Construction land quota, first-price auction, second-price auction, preference

#### I. INTRODUCTION

The rapid development of industrialization and urbanization in China has led to a serious shortage of urban construction land quotas, while the circulation of rural construction land is restricted and largely idle. Therefore, in 2008, Chongqing took the lead in setting up a rural land exchange to carry out the experiment of urban and rural construction land quota trading. The essence of the construction land quota is to convert the spatial attributes of the land, replace the cultivated land in remote areas with urban and rural construction land. The purchased construction land quota can be included in the new construction land plan to increase the same amount of urban construction land, and the quota winner can use it to carry out its investment projects in advance, which has high market value [1]. At present, in the practice of construction land quota trading, it is impossible to fully reveal the true value of the quota based on its acquisition cost for pricing guidance. Therefore, improving the auction system of construction land quotas and protecting the income of farmers is an urgent problem to be solved in the formulation of the quota system.

Some domestic scholars believe that the quota is essentially the transfer and transaction of land development rights, and the quota cost price is determined by the production price plus the compensation price of rural land development rights [2], [3]. Wang and Yang [4] pointed out that the quota price in Chongqing includes two parts: the offset cost and the selection price that developers are willing to pay. Cheng [5] emphasized the financial attribute of quota trading, based on the asset trading of similar securities carried out by specific securities intermediaries. Zhang [6] analyzed the causes of Chengdu's quota price, including opportunity cost, non-cost factors brought by the 'license access' system and speculative factors brought by the transfer. Wen and Zhang [7] pointed out through the land development right transfer model that the smaller the total amount of development, the higher the transfer price of development right. Some scholars have studied quota pricing from the perspective of game theory [8], [9]. He [10] analyzed the transaction costs and benefits of farmers, governments and developers, and determined the quota pricing method. Zhang and Wang [11] discussed the game between farmers and the government, analyzed the cost and income constraints of developers, found the shortcomings of the current quota pricing model, and proposed solutions. The quota system has rich connotation. In addition to the production cost, the quota value also includes the important values derived from the quota transaction system, such as the right to choose and the right to land development. The realization of the derivative value depends on the quota holder's acquisition of the development right in the subsequent construction land auction, which is the quota use process. The above research on quota pricing lacks the overall consideration of quota auction and use process. Lu [12] standardized the use of quotas by building a mathematical model for developers to participate in quotas and land bidding. The quota auction and subsequent construction land auction are regarded as two stages of sequential auction for research, which can more accurately reveal the real value of quotas. Therefore, this paper mainly uses the sequential auction theory for reference to study the quota auction pricing.

Yin and Wang [13] constructed a sequential auction model and solved the optimal auction mechanism in the case of two types of goods. Wang, Yang and Zhang [14] constructed a stochastic equivalent sequential second-price auction to study the impact of quantity discounts on buyers' balanced bidding strategy and expected surplus. Sørensen [15] studied the stochastic equivalent sequential auction and pointed out that the expected equilibrium price does not necessarily show a downward trend in the middle of the auction process. Shin [16] considers two items to be auctioned at the first price. Each auction has two bidders. The agent observes the private value of the auction items in advance, confirming the existence of pure strategy Bayesian Nash equilibrium. Crisps and Ireland [17] consider that sellers sell two items to potential buyers who are not related to each other, and study the optimal auction mechanism of sellers under the circumstances of possible external influence. Jofre-Bonnet and Pesendorfer [18] studied the sequential first-price and sequential public price-raising procurement auctions. Buyers tend to adopt the form of public price-raising auctions, while bidders hope to

Manuscript received February 28, 2023; revised March 29, 2023; accepted April 12, 2023.

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adopt the form of first-price auctions. Donna and Esp ń-S ánchez [19] studied the sequential auction between water rights of units, and found that the bidders paid a higher price for the first unit of water rights, in order to contain other bidders in the subsequent unit of water rights auction, and finally win the auction of all subsequent units of water rights.

Based on the practice of quota auction, considering the preference of construction land location, this paper constructs the quota auction and the construction land auction in the implementation phase into a two-stage sequential auction model for research, breaking through the previous quota system research scope. This paper studies the quota pricing from a quantitative perspective, analyzes the influence of preference on the quota auction strategy, and derives the sequential first-price and sequential second-price auction equilibrium strategy. This paper compares the quota price of the two auction forms and the expected income of farmers, and uses an example to verify the conclusion, in order to try to make some efforts on the pricing of the quota auction in China.

#### II. THEORETICAL MODEL

#### A. Problem and Assumption

According to the quota system of construction land in China, the local government sells two items of construction land quota and construction land through auction. In addition, the system stipulates that the bidders that obtain the construction land quota has the right to selection for the construction land. Bidders can choose a plot of land among all the undeveloped construction land in the city, and the selected construction land will be auctioned in the second phase. The use of selection makes the bidders have more preference for construction land, that is, the bidders believe that they can obtain excess income in the proposed development project. Therefore, this paper establishes a sequential auction of two types of items on construction land quota and construction land. It is assumed that the risk-neutral government will sell two items by holding two consecutive stages of auction. The first stage is the construction land quota auction (Item 1), and the second stage is the construction land auction (Item 2), and the value of Item 1 and Item 2 are not related. The bidder  $i \in \{1,2\}$  has an effective use function  $u: R_{\perp} \rightarrow R$  to satisfy u(0) = 0, u' > 0, u'' < 0. Each bidder pursues maximum utility and has independent private value. It may be assumed that the bidder  $i \in \{1,2\}$  evaluates item 1 as  $v_{1i}$  and item 2 as  $v_{2i}$ , which is subject to uniform distribution on  $[0, \omega_i]$ , distribution function  $F(\cdot)$  and density function  $f_i(\cdot)$ .

There are two risk-neutral bidders participating in the two-stage auction at the same time, assuming that the winner of the first-stage auction is called the leader (L) and the loser is called the follower (F). The leader holds item 1, and can select the corresponding cultivated land in line with the urban and rural planning and land planning as his own construction project. The seller will convert the cultivated land selected by the leader into urban construction land, namely, item 2, and hold the second stage of item auction. Based on the selection

system, leaders can select preferred construction land, which is auctioned in the second phase. This preference increases the valuation of the leader. The preference coefficient is set to  $r, r \ge 1$ , and the utility of item 2 to the leader is  $U_L(v_L) = rU_i(v_i)$ . The utility  $U_F(v_F)$  of item 2 for followers remains unchanged, and there is  $\omega_L = r\omega_i$ . The highest bidder in each stage wins the item in that stage. Suppose that if the follower wins the second stage of item auction, the leader will be paid the price of item 1 in full, and item 1 will be obtained at the same time, and the auction will end.

#### *B. Bidders' Equilibrium Bidding Price with First-Price Auction*

According to the quota of construction land and the actual situation of construction land in China, we consider two stages of sequential auction with the first-price. Equilibrium satisfying sequential rationality has the following properties: given any result of the first auction, the strategies in the second auction form an equilibrium [20]. Therefore, the recursive method can be used to solve the balanced bidding strategy of bidders. We can get the following proposition 1.

Proposition 1: In the first-price sequential auction with preference, the bidders' equilibrium bidding price in the first and second stages are:

$$\begin{split} \beta_{1i}^{I}(v_{1i}) &= \frac{1}{2} v_{1i} + \Delta^{I}(r) , \\ \begin{cases} \beta_{2L}^{I}(v_{2L}) &= \frac{1 - \sqrt{1 - k_{L}(rv_{2L})^{2}}}{k_{L}rv_{2L}}, k_{L} = \frac{1}{r^{2}\omega_{2L}^{2}} - \frac{1}{\omega_{2F}^{2}} , \\ \beta_{2F}^{I}(v_{2F}) &= \frac{1 - \sqrt{1 - k_{F}v_{2F}^{2}}}{k_{F}v_{2F}}, k_{F} = \frac{1}{\omega_{2F}^{2}} - \frac{1}{r^{2}\omega_{2L}^{2}} , \\ \beta_{2F}^{I}(v_{2F}) &= \frac{1 - \sqrt{1 - k_{F}v_{2F}^{2}}}{k_{F}v_{2F}}, k_{F} = \frac{1}{\omega_{2F}^{2}} - \frac{1}{r^{2}\omega_{2L}^{2}} , \\ \beta_{2F}^{I}(v_{2F}) &= \frac{1 - \sqrt{1 - k_{F}v_{2F}^{2}}}{k_{F}v_{2F}}, k_{F} = \frac{1}{\omega_{2F}^{2}} - \frac{1}{r^{2}\omega_{2L}^{2}} , \\ \beta_{2F}^{I}(v_{2F}) &= \frac{1 - \sqrt{1 - k_{F}v_{2F}^{2}}}{k_{F}v_{2F}}, k_{F} = \frac{1}{\omega_{2F}^{2}} - \frac{1}{r^{2}\omega_{2L}^{2}} , \\ \beta_{2F}^{I}(v_{2F}) &= \frac{1 - \sqrt{1 - k_{F}v_{2F}^{2}}}{k_{F}v_{2F}}, k_{F} = \frac{1}{\omega_{2F}^{2}} - \frac{1}{r^{2}\omega_{2L}^{2}} , \\ \beta_{2F}^{I}(v_{2F}) &= \frac{1 - \sqrt{1 - k_{F}v_{2F}^{2}}}{k_{F}v_{2F}}, k_{F} = \frac{1}{\omega_{2F}^{2}} - \frac{1}{r^{2}\omega_{2L}^{2}} , \\ \beta_{2F}^{I}(v_{2F}) &= \frac{1 - \sqrt{1 - k_{F}v_{2F}^{2}}}{k_{F}v_{2F}}, k_{F} = \frac{1}{\omega_{2F}^{2}} - \frac{1}{r^{2}\omega_{2L}^{2}} , \\ \beta_{2F}^{I}(v_{2F}) &= \frac{1 - \sqrt{1 - k_{F}v_{2F}^{2}}}{k_{F}v_{2F}}, k_{F} = \frac{1}{\omega_{2F}^{2}} - \frac{1}{r^{2}\omega_{2L}^{2}} , \\ \beta_{2F}^{I}(v_{2F}) &= \frac{1 - \sqrt{1 - k_{F}v_{2F}^{2}}}{k_{F}v_{2F}}, k_{F} = \frac{1}{\omega_{2F}^{2}} - \frac{1}{r^{2}\omega_{2L}^{2}}, \\ \beta_{2F}^{I}(v_{2F}) &= \frac{1 - \sqrt{1 - k_{F}v_{2F}^{2}}}{k_{F}v_{2F}}, k_{F} = \frac{1}{\omega_{2F}^{2}} , \\ \beta_{2F}^{I}(v_{2F}) &= \frac{1 - \sqrt{1 - k_{F}v_{2F}^{2}}}{k_{F}v_{2F}}, k_{F} = \frac{1}{\omega_{2F}^{2}} , \\ \beta_{2F}^{I}(v_{2F}) &= \frac{1 - \sqrt{1 - k_{F}v_{2F}^{2}}}{k_{F}v_{2F}}, k_{F} = \frac{1}{\omega_{2F}^{2}} , \\ \beta_{2F}^{I}(v_{2F}) &= \frac{1 - \sqrt{1 - k_{F}v_{2F}^{2}}}{k_{F}v_{2F}}, k_{F} = \frac{1 - \sqrt{1 - k_{F}v_{2F}^{2}}}{k_{F}v_{2F}}}, k_{F} = \frac{1 - \sqrt{1 -$$

It can be seen from the formula in Proposition 1 that preference improves the bidders' equilibrium bidding price for construction land quota in the first stage. On the one hand, it may be due to the pursuit of selection by bidders, which increases the intensity of bidding. On the other hand, it may also be due to the selection to strengthen the matching degree between construction land quota and construction land to a certain extent.

When r=1, it means that the bidder has not selected the appropriate preferred construction land after winning the construction land quota in the first stage. Therefore, we can get that in the first-price sequential auction without preference, the bidders' equilibrium bidding price in the first and second stages are:

$$\beta_{1i}^{I^{*}}(v_{1i}) = \frac{1}{2} v_{1i} + \Delta^{I}(\omega) ,$$

$$\begin{cases} \beta_{2L}^{I^{*}}(v_{2L}) = \frac{1 - \sqrt{1 - k_{L}^{*} v_{2L}^{2}}}{k_{L}^{*} v_{2L}}, k_{L}^{*} = \frac{1}{\omega_{2L}^{2}} - \frac{1}{\omega_{2F}^{2}} \\ \beta_{2F}^{I^{*}}(v_{2F}) = \frac{1 - \sqrt{1 - k_{F}^{*} v_{2F}^{2}}}{k_{F}^{*} v_{2F}}, k_{F}^{*} = \frac{1}{\omega_{2F}^{2}} - \frac{1}{\omega_{2L}^{2}} , \\ \end{cases}$$

$$\mu(\omega) = \frac{\omega_{L} \omega_{F}}{\omega_{L}^{*} - \omega_{F}^{*}} [\frac{\frac{\omega_{L}^{*}}{\omega_{F}} + \frac{\omega_{F}^{*}}{\omega_{L}}}{2} + \frac{\omega_{L}^{*} \omega_{F} \arcsin h(\frac{\sqrt{\omega_{L}^{2} - \omega_{F}^{2}}}{\omega_{L} \omega_{F}}) + \omega_{F}^{*} \omega_{L} \arcsin (\frac{\sqrt{\omega_{L}^{*} - \omega_{F}^{2}}}{2\sqrt{\omega_{L}^{*} - \omega_{F}^{*}}} - \omega_{F} - \omega_{L}] .$$

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From the above model, it can be concluded that when there is no influence of preference in the auction process, the equilibrium bidding price in the second stage is consistent. At this time, the bidders' quota equilibrium bidding price is only affected by the bidder's valuation range. When  $\omega_L = \omega_F = 1$ , the bidder's valuation is the same and meets the uniform distribution on [0,1]. At this time, there is  $\beta_{li}^{I*}(v_{li}) = \frac{1}{2}v_{li}^2$ .

# C. Bidders' Equilibrium Bidding Price with Second-Price Sequential Auction

We consider using the second price sealed auction in the two-stage sequential auction of construction land quota and construction land. We use the recursive method to solve the equilibrium bidding strategy of bidders. We can get proposition 2.

Proposition 2: In the second-price sequential auction with preference, the bidders' equilibrium bidding price in the first and second stages are:

$$\beta_i^{II}(v_i) = v_i + \Delta(r), \quad \Delta^{II}(r) = \frac{1}{6} \left( \frac{r^2 \omega_{\scriptscriptstyle L}^2}{\omega_{\scriptscriptstyle F}} - \frac{\omega_{\scriptscriptstyle F}^2}{r \omega_{\scriptscriptstyle L}} \right),$$
$$\begin{cases} \beta_L^{II}(v_L) = r v_L \\ \beta_F^{II}(v_F) = v_F \end{cases}$$

It can be seen from proposition 3 that in the auction process of construction land quota and construction land, the preference has improved the bidders' equilibrium bidding price for construction land quota in the first stage, and also improved the leader's equilibrium bidding price for construction land in the second stage. That is to say, when the construction land is the preferred land of the bidder, the bidder believes that it will obtain excess income in the proposed development project, therefore, the higher the offer. The specific performance is that the bidder uses the selection before the auction of construction land in the second stage. The bidder selects its preferred construction land, and the utility of the bidder increases. At this time, the bidding price also increases.

When r = 1, it means that the bidder has no preference for the land, and the bidder believes that it has obtained average income in the proposed development project. Through calculation, we can get that in the second stage of auction, the bidding strategy of bidder *i* is  $\beta_i^n = v_i$ . In the first stage, the bidders of the two auctions are symmetrical. It is easy to prove that the symmetric Nash equilibrium strategy is  $\beta_i^n(v_i) = v_i + \Delta^n(\omega)$ . Therefore, we can get that in the second-price sequential auction without preference, the bidders' equilibrium bidding price in the first and second stages are:

$$\beta_{1i}^{II*}(v_{1i}) = v_{1i} + \Delta^{II}(\omega) , \quad \Delta^{II}(\omega) = \frac{1}{6} \left( \frac{\omega_{L}^{2}}{\omega_{F}} - \frac{\omega_{r}^{2}}{\omega_{L}} \right)$$
$$\beta_{2i}^{II*} = v_{2i} .$$

In the two-stage sequential second-price sealed auction, when the auction of construction land in the second stage is not affected by preference, the bidder's valuation range has a positive effect on the bidding strategy in the first stage, and when  $\omega_1 = \omega_2 = 1$ , there is  $\beta_{1i}^{II*}(v_i) = v_i$ .

#### D. Farmers' Expected Income

According to the quota system of construction land in China, the local government sells two items of construction land quota and construction land. The local government gives the income of construction land quota to farmers. we can get proposition 3.

Proposition 3: In the sequential auction with preference, farmers' expected income in the first-price and the second-price are respectively:

$$E^{I}[R] = \frac{1}{3}\omega_{i} + \frac{r^{2}\omega_{i}\omega_{F}}{r^{2}\omega_{i}^{2} - \omega_{r}^{2}} \left[ \frac{r\omega_{i}^{2} + \frac{\omega_{r}^{2}}{\omega_{L}}}{2} + \frac{\omega_{i}^{2}\omega_{F} \arcsin h(\frac{\sqrt{r^{2}\omega_{i}^{2} - \omega_{r}^{2}}}{\omega_{L}\omega_{F}}) + r^{2}\omega_{r}^{2}\omega_{L} \arcsin \left(\frac{\sqrt{r^{2}\omega_{i}^{2} - \omega_{r}^{2}}}{r\omega_{L}\omega_{F}}\right) - r\omega_{F} - \omega_{L}\right],$$

$$E^{II}[R] = \frac{1}{3}\omega_{i} + \frac{1}{6}\left(\frac{r^{2}\omega_{L}^{2}}{\omega_{F}} - \frac{\omega_{r}^{2}}{r\omega_{L}}\right).$$

From proposition 3, we can see that in the first-price sequential auction with preference, the factors that affect the farmers' expected income include the preference of the bidders and the maximum value of the goods of the bidders. And in the second-price sequential auction with preference, the preference of bidders and the bidder's maximum valuation of goods affect the farmers' expected income.

Through the above model deduction, we have obtained the equilibrium bidding price and farmers' expected income of the construction land quota and the construction land under the first-price sequential auction and the second-price sequential auction, respectively. Through the analysis of the model, it is found that the land preference of bidders has an impact on the equilibrium bidding of construction land quota and the farmers' expected income. Therefore, we can get the proposition 4.

Proposition 4: Preference increases the equilibrium bidding price of construction land quota and the farmers' expected income. Compared with the second-price sequential auction, the preference in the first-price sequential auction has a greater impact.

Proposition 4 shows that the quota equilibrium bidding price and farmers' expected income increase with the increase of preference coefficient. The reason is that the local government stipulates that bidders with quota can choose their preferred construction land and auction the construction land in the second stage, therefore, the value of the quota is reflected in the additional income obtained by the bidders who choose the land more suitable for the proposed development project. Different bidders have different preferences, therefore, in the auction of quota in the first stage, bidders pay more for obtaining quota. In addition, we also found that in the first-price sequential auction, preference has a greater impact on the quota equilibrium bidding price and farmers' income than the second price.

#### III. NUMERICAL ANALYSIS

To further demonstrate the influence of preferences in Propositions 1 and 3 on the equilibrium bidding price of the first-price and the second-price sequential auction. According to the model in Propositions 1 and 2, we take the quota of construction land purchased by an enterprise as an example [21]. For ease of analysis, maximize the valuation of the business to 1, let  $\omega_L = 1, \omega_F = 1$ , and carry out numerical analysis on the equilibrium bidding price of construction land quota. According to the Land Administration Law of the People's Republic of China, undeveloped land is fined 10%-20% of the land price. In order not to lose the generality, using Matlab for numerical analysis, we limit the value range of *r* to [1,3], as shown in Fig. 1.



Fig. 1. Impact of preference on equilibrium bidding price of quota.

Fig. 1 clearly shows that whether it is the first-price sequential auction or the second-price sequential auction, the preference has a positive impact on the equilibrium bidding price of quota. The greater the influence of preference on bidders, the higher the bidding price of quota. And as can be seen from Fig. 1, the range of bidding price in the first-price sequential auction is [0.2,0.5], and the range of bidding price in the second price sequential auction is [0.4,1.8], which means that preference has a greater impact in the second-price sequential auction, and with the increase of preference, the difference between the first-price and second-price is larger.

When there is preference, the bidders' valuation of the construction land quota increases. Therefore, in order to select a preferred construction land, bidders are willing to pay a higher quota bidding price in the first stage. The increased quota bidding price is reflected in the increased matching degree between the construction land and the proposed development project in the second stage. Therefore, the excess profit of the construction land preferred by the bidders in the proposed development project is the added value of the equilibrium bidding price of the construction land quota. In addition, in the case of preference, bidders bid higher in the second price sequential auction. Therefore, for the government, the first is to implement the construction land quota selection, and the second is to choose the second-price auction to sell the construction land quota and construction land, which is conducive to improving the auction income of the construction land quota.

In order to further demonstrate the influence of the preference in Proposition 4 on the expected income of farmers. We analyze according to the model in proposition 3, as shown in Fig. 2.

Fig. 2 clearly shows that in the first-price auction and the second-price auction, there is a positive correlation between the preference of bidders and the expected income of farmers. Compared with the first-price sequential auction, the

preference in the second-price sequential auction has a greater impact on farmers' expected income. Proposition 4 is proved. Therefore, farmers prefer the government to use the second price auction to sell the construction land quota and construction land to obtain more income.



Fig. 2. Impact of preference on farmers' expected income.

### IV. CONCLUSION

Based on the characteristics of bidders' preference and the sequential auction theory, this paper constructs the equilibrium bidding model of the first-price and the second-price construction land quota and the farmers' expected income model, and obtains the optimal bidding strategy of the construction land indicators, and carries out a numerical comparison and validation study. The study found that the preference improved the equilibrium bidding price of the construction land quota and the expected income of farmers. The reason for the increase was that the enterprises believed that the construction land with preference would obtain additional income after the project was planned to be developed, to make up for the over payment price in the construction land quota. Therefore, the existence of the construction land quota selection system also verifies this fact, therefore reflects the rationality of this system. In addition, compared with the first-price sequential auction, the equilibrium bidding price of the quota and farmers' income under the second-price sequential auction are higher. First of all, the government grants the right to select construction land quota and improves the influence of preference on bidders, which is conducive to improving the auction income and efficiency of construction land quota. Secondly, the government's choice of the second-price auction can improve farmers' income, increase farmers' enthusiasm to reclaim their homestead, further assist in poverty alleviation and rural revitalization, and increase the utilization rate of rural idle homestead.

The model in this paper is to study two-stage sequential auction, and each stage only has a single item. However, in practice, there will be a large number of items to be auctioned at each stage. The existing research results show that in the sequential auction of goods, the price of post-auction goods decreases, which has an impact on the auction strategy of construction land quota, which will be the further research direction of this paper.

#### APPENDIX

Proof of Proposition 1. By maximizing the expected

revenue of bidders, we have  $v_i = \frac{F_j(v_j)}{f_j(v_j)(v_i - b_i)}$ . For bidders who respectively, we have  $\frac{d\beta_i^l}{dr} \ge 0$ ,  $\frac{d\beta_i^{ll}}{dr} \ge 0$ ,  $\frac{dE^l[R]}{dr} \ge 0$  and  $\frac{dE^{ll}[R]}{dr} \ge 0$ .

did not win the construction land quota in the first stage, we can get  $v_i = \frac{v_i(v_i - b)}{b^2}$ . Through calculation, we have  $v_i = \frac{2b}{1 + k_i b^2}$ .

We can calculate that the bidding strategy of bidder F in the second stage is  $\beta_{2F}'(v_{2F}) = \frac{1 - \sqrt{1 - k_F v_{2F}^2}}{k_F v_{2F}}$ , where  $k_F = \frac{1}{\omega_F^2} - \frac{1}{r^2 \omega_r^2}$ .

Similarly, we can get that the bidding strategy of the bidder *L* is  $\beta_{2L}^{I}(v_{2L}) = \frac{1 - \sqrt{1 - k_L(rv_{2L})^2}}{k_L rv_{2L}}$  in the second stage, where

 $k_L = \frac{1}{r^2 \omega_L^2} - \frac{1}{\omega_r^2}$ . Therefore, the bidding strategy of bidders in the

first stage is 
$$\beta_{li}^{I}(v_{li}) = \frac{1}{2}v_{li} + \Delta^{I}(r)$$
.

In the second stage, when the winner and the loser bid the same, there is  $\beta_{2L}^{I}(v_{2L}) = \beta_{2F}^{I}(v_{2F}), \quad v_{2L} = \frac{v_{2F}}{r(1 - \sqrt{1 - k_F v_{2F}^2})}$ . Similarly,

we can calculate  $v_{2F} = \frac{rv_{2L}}{\sqrt{1 - k_L (rv_{2L})^2}}$ , therefore, we can calculate

that the value of  $\Delta^{I}(r)$  is

$$\begin{split} \Delta^{l}(r) &= \prod_{\substack{\beta_{2L}^{l}(v_{2L}) > \beta_{2F}^{l}(v_{2T}) > \beta_{2F}^{l}(v_{2T})} [f_{2F}(v_{2F})f_{2L}(v_{2L})dv_{L}^{2}dv_{F}^{2}} \\ &- \prod_{\substack{\beta_{2L}^{l}(v_{2L}) > \beta_{2F}^{l}(v_{2T}) \\ \beta_{2L}^{l}(v_{2L}) > \beta_{2F}^{l}(v_{2T})} [f_{2L}(v_{2L})f_{2F}(v_{2F})dv_{2F}dv_{2L}} \\ &= \frac{r^{2}\omega_{2F}\omega_{2L}}{r^{2}\omega_{2L}^{2}-\omega_{2F}^{2}} [\frac{\omega_{2F}^{2}+\omega_{2L}^{2}}{2} - r\omega_{2F} - \omega_{2L}}{2} \\ &+ \frac{\omega_{2L}^{2}\omega_{2F}\omega_{2F}}{2\omega_{2L}\omega_{2F}} \arctan h(\frac{\sqrt{r^{2}\omega_{2L}^{2}-\omega_{2F}^{2}}}{2\omega_{2L}\omega_{2F}}) + r^{2}\omega_{2F}^{2}\omega_{2L} \arcsin(\frac{\sqrt{r^{2}\omega_{2L}^{2}-\omega_{2F}^{2}}}{r\omega_{2L}\omega_{2F}})}{2\sqrt{r^{2}\omega_{2L}^{2}-\omega_{2F}^{2}}}] \end{split}$$

**Proof of Proposition 2.** In the first stage of auction,  $\Delta^{II}(r)$ is the same for both bidders, we have the Nash equilibrium strategy in the first stage is  $\beta_i^{II}(v_i) = v_i + \Delta^{II}(r)$ .

In the second stage auction of quota and construction land, when the leader and follower offer the same price, there is  $v_F = rv_L$ . We have

$$\Delta^{II}(r) = \prod_{\substack{n_L \ge v_F \\ m_L < v_F}} (rv_L - v_F) f_F(v_F) f_L(v_L) dv_L dv_F - \prod_{\substack{n_L < v_F \\ m_L < v_F}} (v_F - rv_L) f_L(v_L) f_F(v_F) dv_F dv_L$$
$$= \frac{1}{6} \left( \frac{r^2 \omega_L^2}{\omega_F} - \frac{\omega_F^2}{r\omega_L} \right)$$

**Proof of Proposition 3.** Assuming that  $v_i$  follows the uniform distribution on  $[0, \omega_1]$ . we have

$$E[m(v_i)] = \int_0^{\omega_i} m(v_i) f(v_i) dv_i = \int_0^{\omega_i} \frac{v_i [v_i + 2\Delta'(r)]}{2\omega_i^2} dv_i = \frac{1}{6} \omega_1 + \frac{1}{2} \Delta(r).$$

In first-price sealed auction, we have farmers' expected income is  $E'[R] = 2E[m(v_i)] = \frac{1}{3}\omega_1 + \Delta'(r)$ .

Farmers' expected income under the second-price sealed auction. we can calculate that the farmers' expected income is the sum of all bidders' expectations in the early stage, we have 1

$$E''[R] = 2E[m(v_i)] = \frac{1}{3}\omega_1 + \Delta''(r).$$

Proof of Proposition 4. Solve the first derivative of construction land quota and farmers' income preference Then it can be calculated that the influence of preference on

the bidding price of quota and farmers' expected income is  $\Delta\beta = \Delta\beta^{I} - \Delta\beta^{II} \ge 0$  and  $\Delta E[R] = \Delta E^{I}[R] - \Delta E^{II}[R] \ge 0$ , respectively.

### **CONFLICT OF INTEREST**

The authors declare no conflict of interest.

### AUTHOR CONTRIBUTIONS

Deng Yang conducted the research, constructed theoretical model, conducted numerical analysis, and wrote the paper; The author had approved the final version.

#### REFERENCES

- [1] Q. Y. Yang and C. Y. Lu, "An analysis of the land ticket trade system in Chongqing for balancing urban and rural construction allocation," Chin Public Admin, vol. 12, pp. 68-71, Dec. 2011.
- [2] J. Q. Qiu and D. C. Qiu, "Study on the pricing mechanism of land tickets in the Chongqing rural land exchange center," China Land Sci, vol. 25, no. 10, pp. 77-81, Oct. 2011.
- H. M. Liu, C. Liu, Y. W. Sun, K. Q. Wang, W. Liu, and T. Long, [3] "Research on land quota market mechanism in the context of construction land reduction: Taking Shanghai City as an example," China Land Sci, vol. 31, no. 02, pp. 3-10, July 2017.
- [4] S. J. Wang and M. H. Yang, "An analysis of land ticket trade of country homestead use," Fin Econ, vol. 4, pp. 95-101, Apr. 2009.
- S. Y. Cheng, "Land Securities exchange: Evolution of institution and [5] combination of elements," Academic Monthly, vol. 42, pp. 70-77, May. 2010.
- Z. M. Zhang, "An analysis on the Chengdu's "land Ticket" trade [6] price," Res Econ Manag, vol. 07, no. 14, pp. 84-90, Jul. 2012.
- L. J. Wen and A. L. Zhang, "Relationship among institutional [7] innovation of land coupon, market mechanism of land development right and fulfillment of rural land assets value," China Land Sci, vol. 30, no. 7, pp. 33-40. Jul. 2016.
- [8] W. D. Meng and H. Xiong, "The issue research on land ticket price and income distribution based on game theory in Chongqing," Ecological Economy, vol. 31, no. 04, pp. 20-23, Apr. 2015.
- [9] Z. L. Ma and J. F. Li, "Analysis and research of Chengdu's land tickets trading system based on game theory," World Sci-Tech R & D, vol. 35, no. 2, pp. 265-268, Feb. 2013.
- [10] X. F. He, "Farmer interest, cultivable land conservation and land levying system reform," J Nanjing Agric Univ (Soc Sci Ed), vol. 12, no. 4, pp. 1–5, Apr. 2012.
- [11] P. Zhang and Q. Wang, "Analysis on the intrinsic value and pricing mode of land ticket transaction--Taking Chongqing as an example,' Rural Econ, vol. 05, pp. 39-45, May 2017.
- [12] L. Lu, "Problems in the landing link of the land ticket system quota and their improvement path-Based on the model of developers participating in land ticket and land auctions," Manag World, vol. 10, pp. 178-179, Oct. 2016.
- [13] H. Yin and X. J. Wang, "Optimal auction mechanism for complements," Systems Engineering-Theory & Practice, vol. 26, no. 9, pp. 60-65, Sep. 2006.
- [14] X. J. Wang, S. Yang, and L. B. Zhang, "Research on quantity discount in sequential combinatorial auctions with stochastically equivalent complementary objects," Syst Eng Theor Pract, vol. 34, pp. 1196-1201, Aug. 2014.
- [15] S. T. Sørensen, "Sequential auctions for stochastically equivalent complementary objects," Economics Letters, vol. 91, pp. 337-342, June 2006.
- [16] W. Shin, "Simultaneous auctions for complementary goods," Trading and Market Microstructure, vol. 12, pp. 1-10, 2013.
- M. Cripps and N. Ireland, "Optimal complementary auctions," Bulletin [17] of Economic Research, vol. 53, no. 2, pp. 81-100, 2001.
- [18] M. Jofre-Bonet and M. Pesendorfer, "Optimal sequential auctions," International Journal of Industrial Organization, vol. 33, pp. 61-71, March 2014.
- J. D. Donna and J. A. Esp ns ánchez, "Complements and substitutes in [19] sequential auctions: The case of water auctions," Rand Journal of Economics, vol. 49, pp. 1756-2171, Mar. 2018.

- [20] V. Krishna, *Auction Theory*, Academic press, 2009.
  [21] Y. Deng, W. D. Meng, B. Huang, and J. Y. Liu, "Auction mechanism on construction land quota with selection on land location," *Plos one*, vol. 17, no. 1, e0263075, January 2022.

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