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Application of Fuzzy Mathematical Model in Assets-Liabilities

Manoj Kumar Jain [1], A. K. Dalela [2] and Sandeep Kumar Tiwari [3]

***Abstract*—The literature on fuzzy logic treats it very much as a new concept, distinct from that of probability logic, even though both ascribe to event numbers in interval [0, 1]. An individual financial planning is a decision problem faced by an individual whose aim is to manage his consumption and investment decisions to achieve a real set of financial targets given by his current and expected income, over a long term horizon. Various financial systems in the corporate as well as individual context are under pinned by a cash flow balancing movement. A basic aspect of financial planning encompasses such matching behavior of cash flows and is given by the generic label-Asset and Liability Management (ALM). From a mathematical perspective above models can be set up in an educational form involving non-negative variables which represent inflow and outflow of funds and put back of retained assets and funds from one planning period to the next planning period [1, 5, and 7].**

ALM has emerged as an ideal framework to address this type of decision problem under uncertainty, in which the achievement of a strategic objective is made conditional on the effective management of assets and liabilities over time. The individual problem can be regarded as an extension of a personal savings consumption model with a limited number of savings opportunities and a rich set of individual and regulatory constraints with a long-term objective. The peculiarity of the individual ALM problem comes from the extent and implications of a modeling approach, in which, principle is expected to capture the different features of the management of a financial position with a typically long-term horizon, up to and sometimes beyond retirement for an investor whose preferences may very well change over the planning horizon [1, 9].

An ALM model for controlling risk of underfunding is presented in this paper. The basic model involves multi period decisions (portfolio rebalancing) and deals with the usual uncertainty of investment returns and future liabilities for pensioner. Therefore it is well-suited to a fuzzy programming approach. A fuzzy programming dominance concept is applied to measure (and control) risk of underfunding of pension Asset and Liability.
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***Index Terms*—PV-Present Value ALM – Asset- liability Management**

I. Introduction

In financial sector like bank, insurance company etc, asset and liability management is the practice of managing risks that arise due to uncertainty happened between the assets and liabilities (debts and assets) of the financial institution.

[1]School of Studies in Mathematics, Vikram University, Ujjain. M.P, INDIA, Email: Manoj.kumarjain@gmail.com
[2]Department of Mathematics, Govt Science College, Jabalpur, M. P., INDIA, Email: dalela@ymail.com
[3]School of Studies in Mathematics, Vikram University, Ujjain. M..P., INDIA, Email: skt_tiwari75@yahoo.co.in

Financial institutions face several risks such as the liquidity risk, interest rate risk, credit risk and operational risk. ALM is a strategic management instrument to manage interest rate risk and liquidity risk faced by financial institution, other financial services companies and corporations [6, 8].Financial institution manages the risks of Asset liability mismatch by matching the assets and liabilities according to the maturity pattern or the matching the duration, by hedging and by securitization.

Modern risk management now takes place from an integrated approach to enterprise risk management that reflects the fact that interest rate risk, credit risk, market risk, and liquidity risk are all correlated. At a personal level typically a young professional may set up savings after child birth as he or she goes through the college systems. The savings are assets suitably invested in bonds and shares and future payment for college fees are liabilities. At a corporate level many institutions take contributions from working employees of a corporation and invest these contributions by acquiring assets [2]. These assets are, however, pledged to meet the pension payments of the persons at future dates of their retirement. These pension payments are again the liabilities for the financial institution. A basic aspect of financial planning encompasses such matching activities of cash flows and is given the basic label: ALM. From a mathematical perspective these models can be set up in an educational form involving non-negative variables which represent inflow and outflow of funds and put back of retained assets and funds from one planning period to the next [2,10, and 12].

So what more can be expected from ALM than the established techniques? To answer this, it is necessary to ascertain the pitfalls and difficulties encountered when making investments decisions. It is important to understand the risks that are borne when investing in a particular security or portfolio of securities. Generally, the higher the risks have undertaken, the higher the possible returns on that investment. But there are other constraints that cannot be ignored such as the nature of uncertainty in the decision process, taxes and transactions costs. There may also be legal guidelines and other policy requirements such as institution-specific rules on asset mix [4].

Returning to the fundamental aspect that any company has both assets and liabilities, it is clear that in the course of business the company will benefit from cash inflows and also have to meet liabilities. When asset streams are greater than liability streams there is a surplus and vice-versa when liability streams are greater than asset streams, there is a deficit (Figure: 1) A company will always try to make sure that there is always a surplus but, in situations where there is a deficit, corrective measures can be taken to protect the company financially in the short-term. In the long term however, a company continuing to accumulate shortfalls is likely to be in a serious financial position and may be on the verge of insolvency). To avoid this financial quagmire, requires advanced and meticulous financial planning, and for large organizations ALM is invaluable [5, 7].

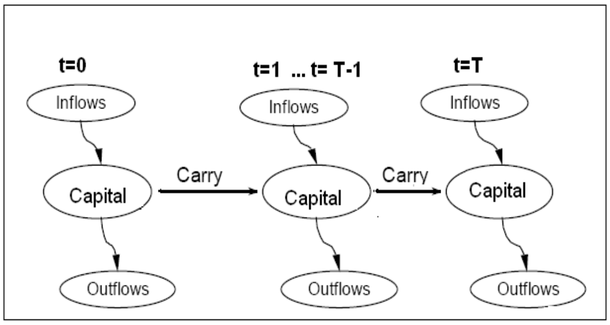

Figure 1: An ALM Fuzzy Programming model

Surplus Wealth= Assets- PV (liabilities) - PV (goals)

Asset allocation decision making is a crucial part of a company's risk management system. Currently, the idea of asset-class investing is becoming more complex than the usual preconceived notion of just investing in equities, fixed income products or cash products. This is mainly due to the fact that hundreds of separate and distinct asset classes can be identified, and still more are flooding the markets. In addition these asset classes have unusual risk and return combinations and their correlations to the other products vary. They reasoned on the premise that only four elements could contribute to investment results: investment policy, individual security selection, market timing and costs. By using a regression analysis, they attributed the contribution (or lack of it) to each of the four elements [6].

Their conclusions were quite astonishing. They found out that the biggest single factor explaining performance was simply the investment policy (asset allocation) decision that determined how much a fund should hold in stocks, bonds or cash. Attempts at market timing most of the time amounted to a reduction in returns, and individual stock selection on average resulted in a reduction to the fund's returns. There was a wider variation in individual stock selection impact than in market timing, and a few managers were able to affect performance during the time period in a positive manner. From this is the importance that of distinguishing between *strategic* and *tactical* asset allocation decisions can be seen [11]. As per my best knowledge; there are not any literatures available for fuzzy mathematical model in field of asset and liabilities. Lot of work has done through stochastic process in field of assets and liabilities

A. Past, Present & Future: A decision-making perspective

There are well-known models such as the Markowitz mean-variance model which has been used to capture uncertainty and make hedged decisions. Unfortunately, it relies entirely on history and makes a single period static decision. The real question that should be asked is: whether history should be taken into account to make future decisions? History does not always repeat. So the answer is that historical data should not be ignored. However, our models should be forward-looking with event trees of future scenarios. The flow of data and processing this into an analytic database and finally use of models which support hedged optimum decisions are in Figure: 2.

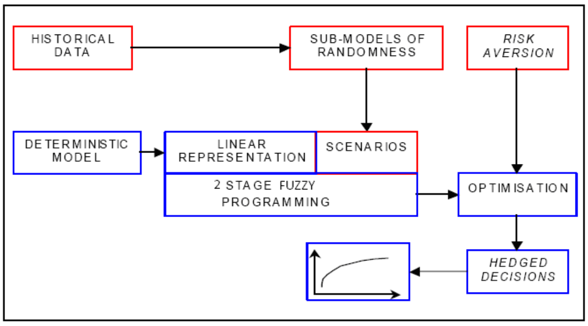

Figure 2: From Historical data to Optimum ALM decisions

II. Risks Faced by Financial Institutions & Risk Quantification

In the finance world, future unpredictability is termed volatility: volatility of asset prices and uncertain liabilities clearly affects financial plans. In general such uncertainties lead to possible financial loss or in other words financial risk. But that does not mean uncertainty equates to or is synonymous with risk. Depending upon the decision-maker or the fund manager's utility, there are many alternative measures of risk.

The choice of an appropriate risk measure that captures an individual's investment preferences has been, and continues to be, the subject of a long debate between academics and practitioners. This is not surprising, since without prior assumptions on the risk preferences of the individuals or the forms of the alternative distributions, it is likely that two individuals will consider risk from alternative perspectives. In general, risk measures can be divided in two groups depending on their perception of risk. The first group contains the so-called dispersion risk measures that quantify risk in terms of probability-weighted dispersion of results around a specific reference point, usually the expected value.

Measures in this category penalize negative as well as positive deviations from a pre-specified target. Two of the most well-known and widely applied risk measures, 1- *variance* or *standard deviation 2- T*he expected or *means absolute deviation.* Regulation of risk is naturally important in the context of banks and financial institutions planning and operations. The risks faced by financial institutions come from different sources of uncertainty. These are then classified accordingly.

Today the following are the accepted areas of risk-operational, credit, liquidity, operational, systemic, political and legal risks. It is a widely accepted notion that financial institutions and more generally banks are in the business of managing risks. The better they manage these risks, the better placed they are in dealing with very rare but possibly commercially destructive events. Moreover, as it is known in the financial markets- A company's reputation is only as good as its last transaction. Hence any let-up in controlling the different aspects of a business could severely dent its future expansion [10].

The following are some of the financial risks that an organization may encounter. By financial risks, broadly means that part of uncertainty that relates to the returns of assets arising from unanticipated and unpredictable events. These events may initiate runs on banks or create a banking panic. In this part, some of the more common risks are discussed. First of all, ***credit risk*** means the risk that arises in the event that counter party defaults on its obligations. The losses can be very substantial for any firm- for example, defaults on mortgage payments or companies' not honoring their bond repayments. Moreover, ***liquidity risks*** *are defined* as an event when it is difficult or expensive to make changes in the composition of one's portfolio. This usually takes place when there are crises in the global markets or following some unexpected political events. Furthermore, ***political risks*** are usually country-specific and relate to the political uncertainties and policies of a particular government- a current example of the existence of political risks could be Zimbabwe where recent events have created some instability and reduced investment in the economy. Finally, we could define a situation where the financial sector has collapsed and where runs on banks are present and problems of liquidity and defaults surface- an apocalyptic situation in a senses ***systemic risk*** *[4, 6]*.

III. UNDERSTANDING RISK IN YOUR ORGANIZATION

Establishing a framework to deal with risk is fundamental. In many cases, a significant cause of failure can be tied directly to an incomplete or absent framework for managing risk. Figure 3 below depicts a process for the development of a risk management framework. The steps identified are aimed at understanding the risks in the business and the importance or danger of each - knowing this is a prerequisite to quantifying the risk appetite of the organization in the context of the business strategy. This 'blueprint' for risk can then be formally merged with the organizational strategy. The outcome of these four steps can then be used to crystallize a overnance and management framework for risk which in-turn can be used to fully develop implementation and management plans [13].

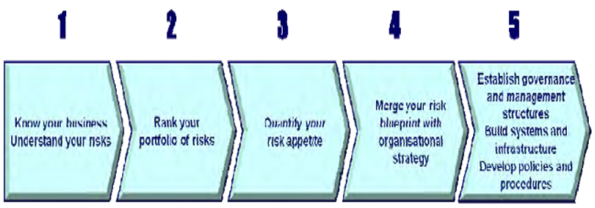

Figure 3: Development Process - Risk Management Framework

A. *Know your business*

Different organizations run different risks and have different risk profiles. Further, just because an organization is in the same industry and/or uses the same products or services does not mean it has the same risk needs. Size does matter and one size does not fit all – a proper diagnostic is required for each organization. For example, consider two small commercial banks with a similar product portfolio operating in different jurisdictions where capital markets development is at different stages. The limitations of the less developed market will mean that 'like' products represent higher risk. Similarly, customer profiles will be distinctly different reflecting inconsistent behaviors and resulting in different default characteristics. Such aspects not only have an impact on weighting the importance of each risk type in the portfolio, they also impact on the framework, operational and infrastructure requirements [10].

B. *Analyze and rank your risks*

Some risks are more difficult to manage than others and a process of assessing the level and importance to each individual organization is necessary. The same risks can represent a higher or lower exposure in different organizations. For example, the recent crisis has highlighted the vastly different risk profile attached to the re-financing risk of a banks' mortgage portfolio versus a non-bank. The strength of a commercial banks deposit franchise, with or without access to credit guarantees, means that the refinancing risk is significantly lower and less costly than that of a non bank, which in contrast tends to be reliant on the securitized markets for funding. This represents a competitive advantage for the banks, particularly in a high stress environment. Clearly the refinancing risk needs to be managed by both organizations; however, a non-bank would rank the importance and difficulty of managing the risk differently than a bank. Another important aspect is that risks within the enterprise are inter-related and cannot be viewed in isolation – understanding these relationships can assist in the identification of natural offsets.

C. *Decide how much risk you should take*

Determining risk appetite is difficult. It is both a quantitative and qualitative process and should be undertaken considering; the current operating environment; the organizations cash-flow; strategy and earnings / balance sheet capacity. A bank should understand its business portfolio and the likely impact, under different scenarios (including high stress), of associated risks on earnings of each segment – both aggregated and disaggregated. The recent global crisis, which has brought a number of organizations previously considered impregnable to their knees, has reinforced the significance of understanding how risk levels change and impact during different operating environments [3].

D. *Make risk part of the organizational fabric*

Risk management is not a part time activity. It is a dynamic process that must be understood and acted on in virtually every part of the organization. The goal of creating a risk management culture is to create a situation where staff and management instinctively look for risks and consider their impacts when making effective decisions. Management of risks is generally a highly fragmented process in most organizations - by involving the enterprise in the process it becomes a less difficult task.

E. *Formalize your framework*

The work in analyzing the organizations risks, ranking risks and quantifying risk appetite is prerequisite to establishing the framework and governance structure. The final components of the framework will depend on the nature of the underlying business - in the case of a bank; Figure 4 below shows the typical components [11, 13].

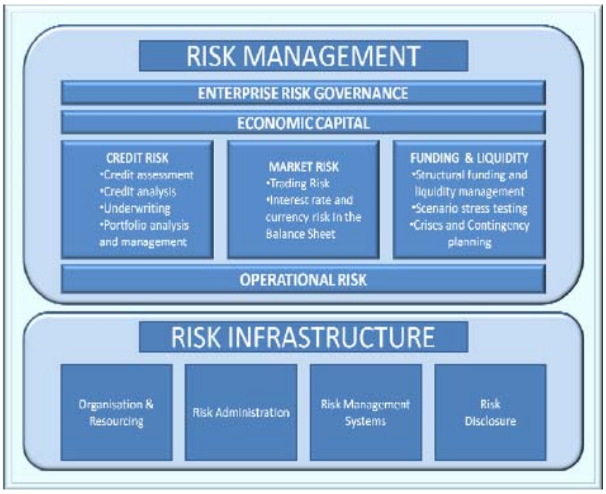

Figure 4 - Generic Bank Risk Management Framework

IV. Fuzzy Programming Model for Pension Asset and Liability

- Two-stage fuzzy programming model with amount of pension bonds held Z_b , sold Y_b and bought X_b and the initial cash C being first stage decision variables. Initial amount of bond held O_b and P_{bT} is the price of bond at time.
- Amount borrowed br_t^f, lent le_t^f and deviation of asset and liability present values (LPV_t^f , $BPV_t^{b,f}$) are the non-implementable Fuzzy decision variables. δ_t^f is fuzzy variable which is depends upon Mark to market (MTM) piece. δ_t^f = {-1,0,1} = {price is decrease, price constant, price increase).
- Multi-objective:

1. Minimise total present value deviations between assets and liabilities
2. Minimise initial cash required

$$Min\sum_{b=0}^{B} LPV_t^f$$

$$Max\sum_{b=0}^{B} BPV_t^{b,f} \qquad \forall b, t = 1..T-1$$

$$Max\sum_{b=0}^{B} P_{bT}\ O_b$$

Fuzzy Programming Model Constraints

1. Cash-Flow Accounting Equation:

$$\sum_{b=1}^{B}(1+\alpha)\delta_t^f x_b \le C + \sum_{b=1}^{B}(1-\alpha)\delta_t^f y_b$$

$$\delta_t^f \in \{-1,1\} \qquad \forall b, t = 1..T-1$$

$$\forall b, t = 1..T-1$$

Where α is the percentage of equities in the investment portfolio.

2. Inventory Balance Equation:

$$z_b = O_b + x_b - y_b$$

3. Present Value Matching of Assets and Liabilities:

$$\sum_{b=1}^{B} BPV_t^{b,f} z_b + \sum_{b=1}^{B} BPV_t^{b,f} x_b + (1+r_t)le_{t-1}^f$$

$$= \qquad \forall f, t = 1..T-1$$

$$devo_t^f - devu_t^f + LPV_t^f + (1+r_t)br_{t-1}^f$$

4. Matching Equations:

$$\sum_{b=1}^{B}(c_1^b z_b + c_1^b x_b) + br_1^f = L_1^f + le_1^f \qquad \forall b$$

$$\sum_{b=1}^{B}(c_t^b z_b + c_t^b x_b) + br_t^f - (1+r_t)br_{t-1}^f =$$

$$L_t^f + le_t^f - (1+r_t)le_{t-1}^f \qquad \forall f, t = 1..T-1$$

$$\sum_{b=1}^{B}(c_T^b z_b + c_T^b x_b) - (1+r_T)br_{T-1}^f = \qquad \forall f$$

$$L_T^f + le_T^f - (1+r_T)le_{T-1}^f$$

5. Non-Anti captivity:

$$br[1,f] = br[1,f1] \qquad \forall f, f1$$

$$le[1,f] = le[1,f1]$$

6. Down Risk Constraint

$$(A_t - Max\sum_{b=0}^{b} P_{bT}\ O_b) / At < R_t \qquad t > 1$$

Where:

- A_t is the predefined target for time period t
- R_t is expressed as a positive fraction and specifies the maximum deviation from the target accepted by the investor

A. Result

The fuzzy scenarios set is presented in the form of Monte Carlo simulations, i.e. 1000 paths were simulated for the trend in investment returns to provide a clear picture of the uncertainty in that respect. In conceptual terms, this means that we in fact assume that the future will manifest itself as one of these 1000 paths. Another familiar simulation technique, as used in Value-at-Risk calculations by banks, is the historical simulation. This describes what would happen if an event or a series of events from the past were to recur. The advantage of this approach is that the results are not influenced by assumptions about distributions, for example.

In the fuzzy simulations we assumed that equity yields are totally independent of past performance. This means that we take no account of the current level of the stock market. Although mean reversion on the stock market is difficult to demonstrate, we can nevertheless assume that the uncertainty of a crash declines if there has recently been a slump in the market. Since the analysis horizon stretches far into the future where pensions are concerned, it is worth investigating the importance of equilibrium recovery on the stock market in more detail. This is done on the basis of a historical simulation, for which we use real equity yields and real interest rates from 1960 to 2003. In the analysis we assume that yields from 2004 will follow the same path as historical yields. For yields from 1960 onwards, this therefore gives a prediction for the period 2004-2047. If we select a later starting point from the 1960-2003 periods, the prediction horizon is correspondingly shorter; for example, using yields during the 2000-2003 periods only offers a prediction up to 2007. Apart from the inclusion of equilibrium tendencies, the historical simulation has the additional advantage that time-varying correlations between interest rates and equity yields are implicitly taken into account.

As regards real interest rates, we use the yields on Dutch long-term government bonds and Dutch inflation. In contrast, the real equity yields are based on an international portfolio spread with the yield being assumed to equal a weighted average of the real equity yields in the various regions. The implicit underlying assumption is that the currency risk is fully hedged, and the costs of the hedge are equal to the inflation differential between the Netherlands and the region in question. As regards the spread, for the 1970 – 2003 period we take the MSCI total return indices for the Netherlands (15%), rest of Europe (35%), the US (40%) and Japan (10%), for the 1965 – 1969 period we take the total return indices for the Netherlands (20%), the UK (30%) and the US (50%), and for 1960 – 1964 we take just the indices for the Netherlands (50%) and the US (50%).

In figure 6(a, b) gives the results for the pension premium and the cover ratio assuming an asset mix containing 50% equities and a fixed actuarial interest rate of 2.75%. These assumptions correspond to the fuzzy scenario in Figure: 5(a, b, c and d)

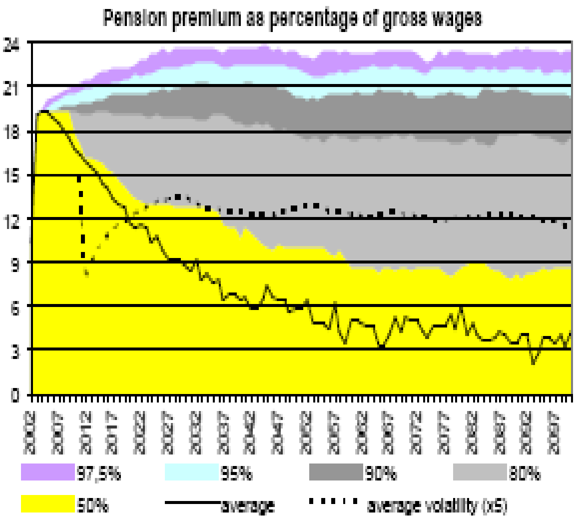

Figure 5a

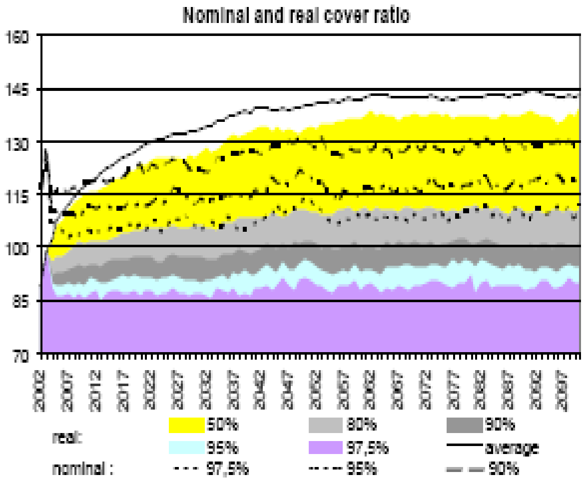

Figure 5b

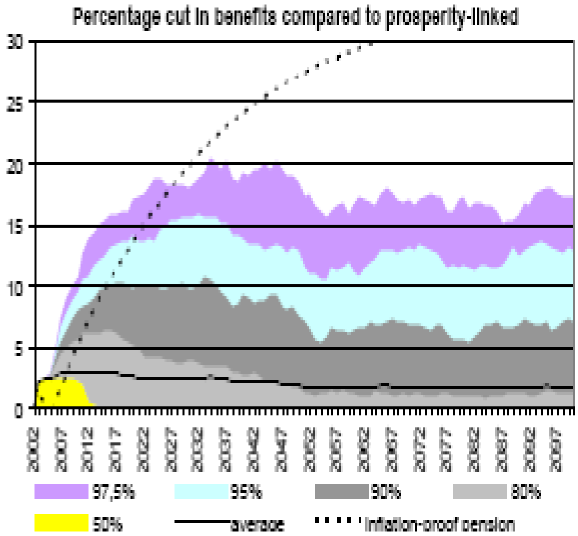

Figure 5c

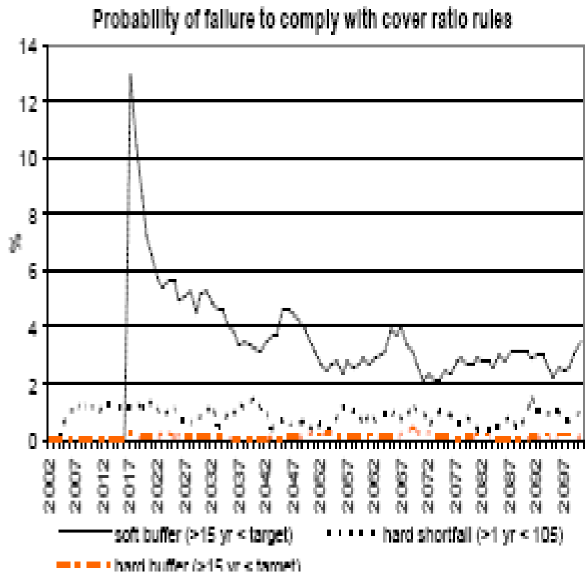

Figure 5d

Figure 5(a, b, c, d): Scenario for 15-year recovery plan with a fixed actuarial interest rate of 2.75%

The maximum pension premium over all 44 historical yield paths is 21.5% of the gross wage bill, somewhat less than the 95%-percentile in the fuzzy scenario. For all scenarios with a starting year between 1960 and 1986, there are substantial premium refunds during the stock market boom corresponding to the late 1990s, up to cumulative 356% of gross wages, without this leading to premiums being charged after the stock market crash years, 2000-2002.

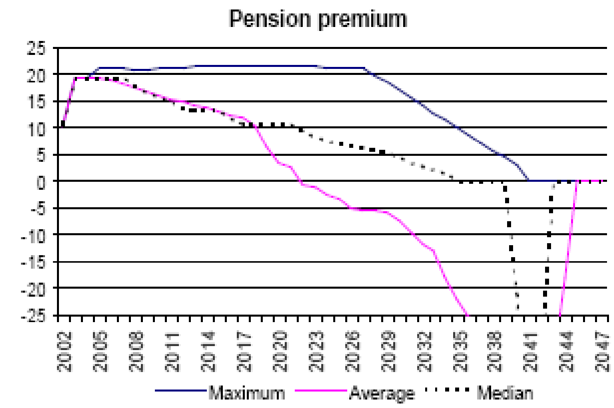

Figure 6a

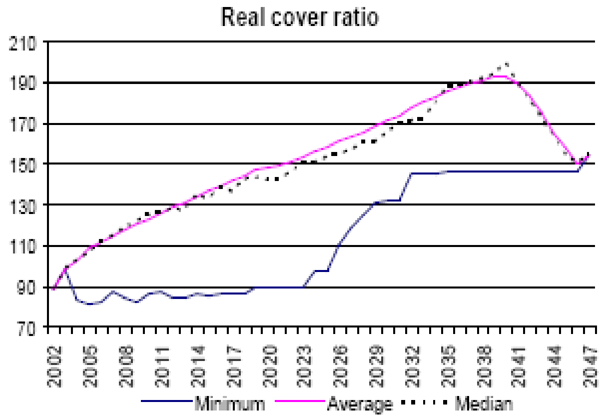

Figure 6b

Figure: 6(a, b) Historical simulations with 50% equities and an actuarial interest rate of 2.75%

By 2008, the average cover ratio is already above the target of 118% and rises steadily thereafter to 190% in 2037. The very high real interest rate and equity yields of the 1980s and 1990s make the pension funds effortlessly rich. Even with the most negative yield paths, the real cover ratio remains just above 82%, comparable to the 97.5% percentile in the fuzzy scenario. Almost every year, the minimum cover ratio relates to a different scenario. Only if future yields correspond to those for the 1969-2003 periods will the cover ratio remain below the target level for longer than 15 years (namely 16 years). One of the reasons for this relatively favorable result is the inclusion of the predicted low wage increase in 2004 (1.4%) and 2005 (0.6%). Also, the positive performance by equities in 2003 (20%) secured a significant improvement in the starting position (cover ratio of 98.5% at the end of 2003 with an actuarial interest rate of 2.75%).

The importance of the mechanisms restoring equilibrium on the stock market is mainly of relevance for the assessment of a variable asset mix. Figure 7(a, b) shows the results of a historical simulation with a variable asset mix. As in the case of the fuzzy simulation Figure 5(a, b, c, d), the percentage of equities is assumed to be 20 until the cover ratio reaches 100%, rising to 50 when the cover ratio equals the target. It is even more apparent than in the fuzzy simulation that adjusting the asset mix in line with the financial position of the fund has hardly any positive impact on the recovery. In the series of poor investment years, 1973-1974 and 2000-2002, it is true that a prompt reduction in the percentage of equities would have initially helped to keep the cover ratio from falling far below 100%, yet the subsequent recovery path is considerably more arduous. Since less advantage is taken of the stock market rally, the cover ratio remains low, making it harder to cope with further negative shocks.

Moreover, the premium setting is significantly higher on average, without being offset by lower premiums at the extremes. A flight-to-quality when cover ratios are low is therefore not very attractive on the basis of either the fuzzy simulation or the historical yield paths.

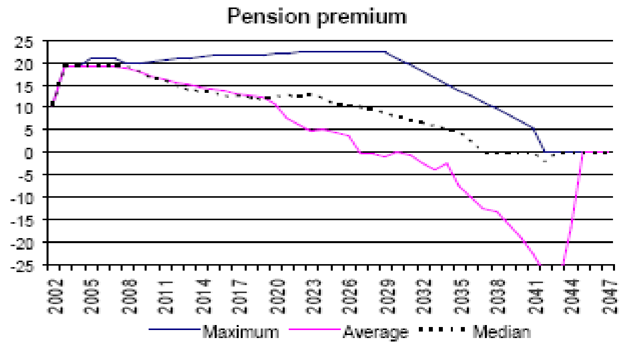

Figure 7a

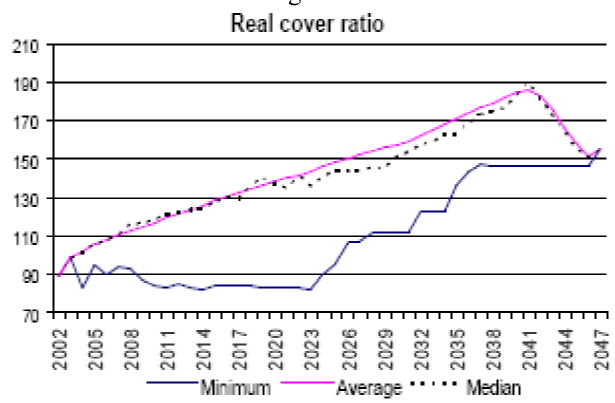

Figure 7b

Figure 7(a, b) Historical simulations with 20 to 50% equities and an actuarial interest rate of 2.75%

V. Conclusion

ALM team is responsible for allocating funding to various lending tool, and for ensuring that the currency, interest rate and maturity sensitivity characteristics of the Bank's assets and liabilities are within prescribed risk parameters. To achieve this, ALM makes extensive use of derivative instruments including currency swaps, interest rate swaps and other interest rate management.

A practical starting point is to construct a fuzzy framework for risk management considering the detail of the business activities the bank is involved in, analyzing and ranking the risks involved in the various businesses and deciding how much risk the bank should take. While the 'headline' categories in the risk management framework will be similar for all banks, the needs in both analysis and management will vary considerably for banks of different sizes and operating in markets of different stages of development. The program to build effective risk management in a bank must be sponsored and overseen by the highest levels of the bank and governance and oversight structures established and embedded in the organizational fabric. This commitment needs to be backed up with a range of additional commitments to ensure budget, technology and people resources are sufficient to execute the implementation plans.

A final, critical point to note is that risk management and ALM are not static activities. Both continue to evolve and new aspects are presented that challenge the organizations capacity. Regular board oversight together with a periodic and detailed review process has to be built into the framework to ensure focus remains appropriate and relevant.

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