Insurance Models for Assessment of the Municipal Losses Due to Natural Disasters

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***Abstract*—Various insurance models for assessment of the possible possible financial losses for a municipality due to natural disasters. Risk situations which describe the possible financial losses of the monitored objects, are considered. The main components of the insurance models are discussed. The loss assessment results can support the municipal government to take more informed decisions for effective use of limited financial resources to activities in emergency situations. A concept for implementing the insurance models as a part of a Web integrated information system for risk management of natural disasters are outlined.**

***Index Terms*—Insurance models, risk situation, loss assessment, municipal losses, natural disasters.**

I. Introduction

In recent years the negative impact of natural disasters on sustainable development of the municipalities increases. Statistic data and scientific research show a growth in number and severity of natural disasters compared to previous years [1]. Billions of dollars cost annual losses resulting from floods, hurricanes, earthquakes, tornadoes, etc. Natural disasters are impossible to avoid, and municipal infrastructure elements cannot be made totally invulnerable. The only viable solution is to prepare towns and communities through a combination of mitigation and adaptation strategies [2],[3].

Hence there is a need to propose models to assess the municipal losses at occurrence of natural disasters. The availability of an adequate assessment of the potential total loss would help the municipal government to take more informed decisions for effective use of limited financial resources to activities in emergency situations. On the other hand is well known that there are various insurance models for loss assessment [4]-[7].

The purpose of the paper is to present various insurance models for assessment of the possible possible financial losses for a municipality due to natural disasters. Some risk situations and assessment models are considered. The average number events and the average severity of losses are estimated. The possible claim amounts are assessed. The proposed insurance model is envisaged to be implemented as a part of a Web Integrated Information System for risk management of natural disasters.

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II. Various Risk Situations

The natural disasters cause various negative consequences of the monitored objects which can be described from a mathematical viewpoint as various risk situations [4],[6].

A. Possible Loss with Fixed Amount for One-year Period

First it is assumed that the suffered total loss is with fixed amount. In particular, the potential loss of one object due to occurrence of one natural disaster within a given period is considered. Thus, if the negative event occurs, the amount of the loss is certain. The potential loss, X , is defined as follows:

$$X = \begin{cases} x & if \quad \eta \\ 0 & if \quad \overline{\eta} \end{cases} \tag{1}$$

where η is the natural disaster (the negative event) which causes the financial loss; x is the amount of the loss (lost severity).

The loss is zero ($X = 0$) when the negative event is not occurred ($\overline{\eta}$).

B. Losses with Random Amounts for One-year Period

Second it is allowed that the loss is with random amount (severity) X . The possible loss realizations can be either discrete variables ($X : 0, x_1, ..., x_{\max}$) or continuous variables ($0 \le X \le x_{\max}$). The outcome $X = 0$ denotes the absence of damage for lack of natural disaster and the outcome $X = x_{\max}$ indicated the realization of the maximum loss amount. The set of discrete potential losses are described as follows:

$$X = \begin{cases} x_1 & if \quad \eta_1 \\ x_2 & if \quad \eta_2 \\ ... & ... \quad ... \\ x_m & if \quad \eta_m \\ 0 & if \quad \overline{\eta} \end{cases} \tag{2}$$

where x_i , $i = 1,2,..., m$ is the *i*-th severity of the loss due to occurrence of η_i - the *i*-th variety of the natural disaster.

The financial loss is zero ($X = 0$) when the natural disaster is not occurred ($\overline{\eta}$).

The negative events $\eta_1, \eta_2, ..., \eta_m$ (accidents) are scaled according to increasing severity of the consequences in terms of amount of the loss. The event η describes completely the natural disaster and it can be represented as the following union

$$\eta = \eta_1 \cup \eta_2 \cup ... \cup \eta_m \tag{3}$$

whereas $\overline{\eta}$ still represents the absence of the negative event.

C. Random Number of Events Each with Deterministic Loss for One-year Period

In this case a random number of events (accidents due to natural disaster) may occur within the stated period and each event implying a deterministic loss. The time horizon is one year.

If random variable Z denotes the random number of accident due to natural disaster within a given year then the total loss is described by

$$X = BZ \tag{4}$$

where B is the amount of the loss for each object; n – the number of the monitored objects, exposed to the risk of the natural disaster; the possible outcomes of Z (the number of potential accidents) are $0,1,...,n$, so that the corresponding outcomes of X are $0, B,..., nB$.

The expected value, $\mathrm{E}X$, of the total financial loss is

$$\mathrm{E}(X) = B.\mathrm{E}(Z) \tag{5}$$

If for the *j*-th monitored object, $j = 1,2,..., n$, the amount of the loss is B_j then, the total loss does not depend on the number of accidents only, as it also depends on which objects affect the natural disaster.

In formal terms, with reference to *j*-th object the random loss amount X_j is defined as follows

$$X_j = \begin{cases} B_j & \text{in the case of accident} \\ 0 & \text{otherwise} \end{cases} \tag{6}$$

Then, the total financial loss is given by

$$X = \sum_{j=1}^{n} X_j \tag{7}$$

It is necessary to mention that the risk, which leads to the random total loss X , is actually a set of individual risks, each one represented by the related loss is B_j (or B).

D. Random Number of Events Each with Deterministic Loss for Multi-year Period

In this case a random number of accidents due to natural disaster may occur for the multi-year period and each event implying a deterministic loss. The time-value of money cannot be disregarded when a longer time horizon is addressed.

It is assumed that the time horizon consists of m years. In particular, it is interested in setting $m > 1$ (for example, $m = 3$, $m = 5$ or $m = 10$). It is still assumed more that the number of the monitored objects is n . Moreover, it is supposed that each monitored object who suffered an accident implying permanent loss any given year is replaced, at the beginning of the following year, by another object. Further new objects are not allowed. Hence, *n* objects are exposed to risk at the beginning of each year.

The individual loss, at the end of the year in which the accident occurs, is B , whatever the year may be (within the stated period). The random loss amount at time *t*, exactly at the end of year *t* is given by

$$X_t = B.Z_t \tag{8}$$

where Z_t is the random number of accidents occurring in the various years for $t = 1,2,..., m$.

It is necessary to note that if the total financial loss as following sum

$$X = X_1 + X_2 + ... + X_m \tag{9}$$

then the time-value of the money is disregard, i.e. a zero interest is assumed.

E. Random Number of Events with Random Loss for One-year Period

It is assumed that a monitored object can be damaged one or more times within the stated period (one-year period), by natural disaster. In each occurrence of natural disaster, the amount of the related damage (financial loss) is random.

In this case the randomness of the loss and random number of negative events are merged together.

In formal terms, it is defined the random number N as the number of occurrences of the natural disaster within the stated period. Then, it is denoted with X_k the damage (loss severity) caused by the *k*-th natural disaster. Hence, the total random damage X (total loss) is defined as follows:

$$X = \begin{cases} 0 & \text{if} \quad N = 0 \\ X_1 + X_2 + ... + X_N & \text{if} \quad N > 0 \end{cases} \tag{10}$$

where $X_k > 0$ for each $k = 1,2,..., N$, when $N > 0$.

The zero damage ($X = 0$) is described by $N = 0$.

It can be determined $x_{\min} = Min\ (X_k)$ and $x_{\max} = Max\ (X_k)$, for each $k = 1,2,..., N$. In particular, the maximum loss severity, $x_{\max}$, could be the cost of the monitored object. However, it is unlikely that, in the case of multiple occurrence of the natural disaster, each event completely destroys the object (which, in the meanwhile, should have been completely recovered. For this reason, it very important to correctly determine the probabilities of random variables $X_1, X_2,..., X_N$ and N .

In relation to the random variable N , it is essentially assumed that the possible outcomes are all the integer numbers $0,1,2,\ldots$. Conversely, it can accepted a maximum (reasonable) outcome $n_{\max}$, so that the possible outcomes are $0,1,2,\ldots,n_{\max}$.

If it is assumed that all the random amounts (damages or loss severities) X_k, $k=0,1,2,\ldots,n_{\max}$, have the same expected value, i.e.

$$\mathrm{E}(X_1)=\mathrm{E}(X_2)=\ldots=\mathrm{E}(X_k)=\ldots=\mathrm{E}\left(X_{n_{\max}}\right) \quad (11)$$

where $\mathrm{E}(X_k)$ is the expected value of the damage resulting from the *k*-th occurrence, then

$$\mathrm{E}(X)=\mathrm{E}(X_1).\mathrm{E}(N) \quad (12)$$

where $\mathrm{E}(X)$ is the expected value of the total damage (total loss), $\mathrm{E}(N)$ is the expected value of the random number of occurrences of the natural disaster in the given period.

III. Some Risk Assessment Models

Many risk assessment models are known [6],[7]. Here, some models are considered only [4].

A. Model for Assessment of the Possible Loss with Fixed Amount

The random loss with fixed amount (1) is expressed. In the case, the determination of the natural disaster (negative event) probability, η is only required in order to design the risk assessment model.

Let it is denoted this probability as $p=P(\eta)$. The expected value of the potential loss X is then given by

$$\mathrm{E}(X)=x.p$$

and the variance, $\mathrm{D}(X)$ is expressed by

$$\mathrm{D}(X)=x^2.p.(1-p)$$

The standard deviation, $\sigma(X)$ is defined as the square root of the variance:

$$\sigma(X)=\sqrt{\mathrm{D}(X)}=\sqrt{x^2.p.(1-p)}=x\sqrt{p.(1-p)}$$

B. Model for Assessment of the Losses with Random Amounts for One-year Period

The discrete probability distribution of the random variable X, describing the severity of the loss ($X: 0, x_1,\ldots,x_{\max}$) is expressed as follows

$$p_i=P(X=x_i),\qquad i=1,2,\ldots,m,\qquad x_m=x_{\max}$$

and the main constraint $\sum_{i=0}^{m} p_i=P(X=x_i)$ is fulfilled. The expected value of the potential damage X is

$$\mathrm{E}(X)=\sum_{i=0}^{m} x_i.p_i\ ,\quad \text{under constraint} \quad \sum_{i=0}^{m} p_i=1$$

and the variance and the standard deviation are given as

$$\mathrm{D}(X)=\sum_{i=0}^{m}\left(x_i - \mathrm{E}(X)\right)^2.p_i \quad \text{and} \quad \sigma(X)=\sqrt{\mathrm{D}(X)}\,.$$

From condition (3) the complete probability of an accident, regardless of the corresponding severity of the natural disaster, is expressed by

$$p=\mathrm{P}(\eta)=\mathrm{P}(\eta_1\cup\eta_2\cup\ldots\cup\eta_m) \quad \text{and} \quad p=\sum_{i=1}^{m} p_i$$

whereas the probability of not occurring of the natural disaster is $p_0=\mathrm{P}(\overline{\eta})$ and $p_0=1-p$.

According to the theorem of conditional probabilities, the following condition is satisfied

$$\mathrm{P}(X=x_i)=\mathrm{P}(X=x_i|\eta).\mathrm{P}(\eta),\qquad i=1,2,\ldots,m.$$

Then, the conditional probability of the loss severity, when the natural disaster is occurred, is given as

$$\mathrm{P}(X=x_i|\eta)=\frac{\mathrm{P}(X=x_i)}{\mathrm{P}(\eta)}=\frac{p_i}{p}\,,\qquad i=1,2,\ldots,m.$$

and the conditional expected value of the potential damage X is

$$\overline{x}=\mathrm{E}(X|\eta)=\frac{1}{p}.\sum_{i=1}^{m} x_i.p_i\ .$$

C. Risk Model for Assessment of the Random Number of Events

A finite discrete distribution of the random number N is investigated. In practice the Poisson distribution is often used.

As a first step a reasonable maximum outcome, $n_{\max}$ is selected. Then, the following probabilities are assigned

$$\nu_i=P(N=i),\qquad i=0,1,\ldots,n_{\max}.$$

The expected value of the random number, N is

$$\overline{n}=\mathrm{E}(N)=\sum_{i=0}^{n_{\max}} i.\nu_i\ ,$$

and the variance is expressed as follows:

$$\mathrm{D}(N)=\sum_{i=0}^{n_{\max}}(i-\overline{n})^2.\nu_i\ .$$

D. Risk Model for Assessment of the Random Loss for One-year Period

The probability distribution of the total loss and the related typical values are of great interest. The variable X usually represents the random cost referred to the stated period (one year). The total random damage X, (10), is a random sum, since the number N of terms in the summation as well as the individual values of the terms are random variables.

The following probabilistic assumptions about the random variables N and X_k, $k = 0,1,2,...,n_{\max}$ are adopted for calculating the expected value $\mathrm{E}(X)$:

- The random variables X_k are independent of the random number N;
- Whatever the outcome n of N, the random variables $X_1, X_2,..., X_n$ are mutually independent and identically distributed with a common expected value (11).

The expected value of the loss severity, X is described as

$$\mathrm{E}(X) = \sum_{i=0}^{n_{\max}} \nu_i . \mathrm{E}(X|N = i) = \sum_{i=1}^{n_{\max}} \nu_i . \mathrm{E}(X|N = i).$$

$$= \sum_{i=1}^{n_{\max}} \nu_i . \left(\sum_{k=1}^{i} \mathrm{E}(X_k|N = i) \right).$$

From the first assumption follows

$$\mathrm{E}(X_k|N = i) = \mathrm{E}(X_k), \quad \text{for each} \quad k = 0,1,2,..., n_{\max},$$

and taking into account the second assumption and the condition (11) it is obtained

$$\sum_{k=1}^{i} \mathrm{E}(X_k|N = i) = i.\mathrm{E}(X_1)$$

Therefore, the expected value of the loss severity is expressed as

$$\mathrm{E}(X) = \sum_{i=1}^{n_{\max}} \nu_i .i.\mathrm{E}(X_1) = \mathrm{E}(N).\mathrm{E}(X_1).$$

The assumption of independence between the random variables X_k and the random number N is is too idealized. More realistic are the situations in which a very high total number of damages is likely associated to a prevailing number of damages with small amounts.

IV. Loss and Claim Amount Assessment

Therefore, the possible amount of claims must be assessed in order to evaluate the financial status of a municipality with respect to the negative consequences due to occurrence of natural disasters.

Let the random number of claims for one-year period is the variable N. The possible claim numbers are $k = 1,2,\ldots,N$. It is assumed that each claim will cause a random financial loss to the municipal budget. The loss relevant to the claim k is noted by X_k, $k = 1,2,\ldots,N$. The claim amount Y_k for claim k is assessed according to municipal financial conditions.

It is reasonably, $Y_k \le X_k$, to prevent moral hazard. In general terms, the claim amount Y_k is a given function of the loss amount X_k. This function is called the *claim function*.

Under the same municipal conditions, a different claim function could be selected for each claim. Here, it is assumed that the same claim function, f will apply to any claim, i.e.

$$Y_k = f(X_k). \qquad (13)$$

Under the *full compensation* arrangement, the municipality pays in full the loss (negative consequences) due to occurrence of natural disasters. In this case the claim function is defined as follows

$$Y_k = f(X_k) = X_k. \qquad (14)$$

In property insurance, arrangement (14) is known as *full value*, while in liability insurance as *unlimited liability*. A graphical representation is given in Fig. 1.

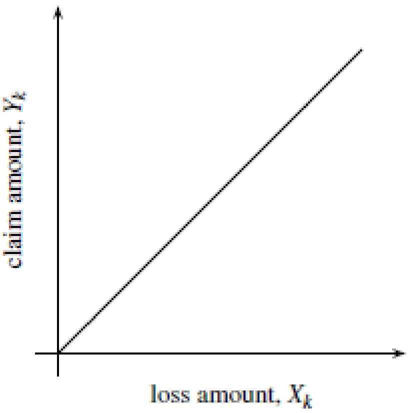

Fig. 1. Claim amount according to full compensation

In the case of property insurance, the maximum loss amount and then the maximum payment by the insurer are given by the value V of the property, $V > 0$. Conversely, no cap is provided for the payment by the insurer in the case of liability insurance, $X_k > 0$ [4].

In this study,if natural disasters have caused material damage, then the maximum loss amount and then the maximum payment by the municipality are given by the value V of the property, $V > 0$. Likewise as in insurance, it is not imposed limits on the payment in the cases of environmental and health damages.

On the other hand, it can be presumed from experience that some extreme values for the amount of loss are unrealistic. Thus, the *maximum probable loss* (or *MPL*), in particular, could be defined as [4]

$$MPL = \inf \{ x : \mathrm{P}(X_k \le x) = 1 \} \qquad (15)$$

Definition: the *MPL* is the highest value for the loss originated by a (single) claim for which the probability to occur is positive. In the case of property insurance, it may turn out $MPL < V$. In the case of liability insurance, it is exclude to observe loss amounts higher than the *MPL*.

Arrangement (14) is clearly unsatisfactory for the insurer as well as for the municipal budget. In this situation, the municipality is not only exposed to the risk of large claims, but it is also facing small claims, which are usually high in numbers and carry processing costs which may exceed the benefit amount.

Further, the victims of natural disasters could be careless in preventing the negative consequences, given that the cost of a claim is fully charged to the municipal budget.

Small claims can be avoided through deductibles. In particular, according to a principle of the minimum deductible the municipality can be intervened only if the loss amount is above a given threshold, the deductible d.

Therefore the claim amount is defined as follows:

$$Y_k = \begin{cases} 0 & if \quad X_k \le d \\ X_k & if \quad X_k > d \end{cases} \tag{16}$$

This claim amount condition (16) is represented in Fig. 2.

According to a *fixed-amount deductible*, an amount d is always charged to the victims of natural disasters (in the insurance to the policyholder). Here, it is evident that if the loss amount is lower than d, there is no payment by the municipal budget (respectively by the insurer). The claim amount is then defined as follows [4];

$$Y_k = \begin{cases} 0 & if \quad X_k \le d \\ X_k - d & if \quad X_k > d \end{cases} \tag{17}$$

The claim amount condition (17) is shown in Fig. 3.

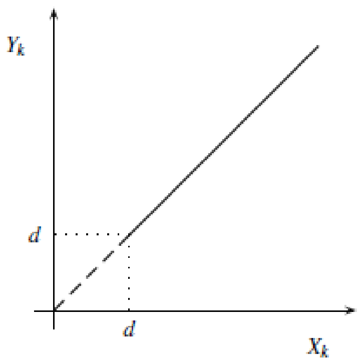

Fig. 2. Claim amount according to the franchise deductible

A proportion α of the loss ($0 \le \alpha < 1$) is charged to the insured under the *proportional* (or *fixed-percentage*) *deductible*. In this case, the claim amount is defined as follows

$$Y_k = (1-\alpha).X_k \qquad \text{for} \qquad 0 \le \alpha < 1. \tag{18}$$

The claim amount according to the proportional deductible (18) is represented in Fig. 4.

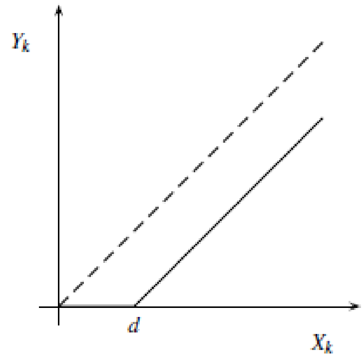

Fig. 3. Claim amount according to the fixed-amount deductible

It must be noted that in the insurance the higher is the loss amount, the higher is the cost charged to the insured.

The arrangement is usual in property insurance, in case the insured value, V', is lower than the current value of the property, V. In this case, the proportion α is given as

$$\alpha = \max\left\{1 - \frac{V'}{V},\ 0\right\} \tag{19}$$

It should also be noted that V is usually ascertained at the time of claim occurrence, while V' is set at policy issue (or renewal time). It is due to a depreciation or a revaluation of the property. It may well turn out $V \le V'$ or $V \ge V'$.

In this study, V' dependent on municipal rules imposed relations of the compensation the negative consequences due to natural disasters.

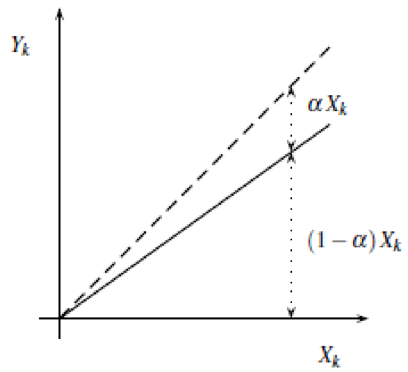

Fig. 4. Claim amount according to the proportional deductible

In the case $V \le V'$, the insurer reduces accordingly the claim amount, to avoid that at issue the insured reports an underestimated value of the property, so to pay a lower premium.

Admittedly, underinsurance (i.e., $V \le V'$) can be a specific choice of the insured. The proportional deductible is applied also in covers where the behavior of the insured can affect the claim cost, such as sickness insurance, theft insurance, all risks motor insurance, and so on.

Usually in order to avoid large claims, the insurer (the municipal government) applies upper limits.

If a *limit value* M is adopted, the claim amount is defined as follows

$$Y_k = \min\{X_k, M\}. \tag{20}$$

In liability insurance, the limit value is also called the *capacity* of the policy. In property insurance (where $M < V$), the arrangement is also called *first loss*. A graphical representation is shown in Fig. 5.

The claim functions described above represent the most common forms of limitation to the insurer's liability. Insurance practice provides further examples of policy conditions. Some of them are in particular suitable for a specific line of business [4].

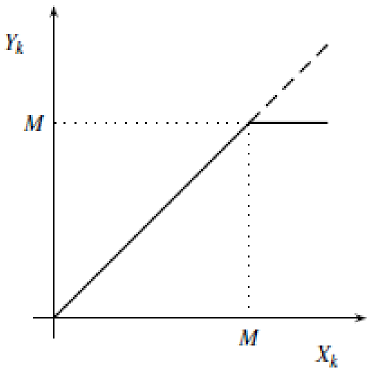

Fig. 5. Claim amount according to the limit value

V. CONCLUSION

Various insurance models for assessment of the possible possible financial losses for a municipality due to natural disasters. Some risk situations and assessment models are considered. Some risk assessment models are considered. A concept for implementing those models in a Web integrated information system for risk management of natural disasters is outlined.

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